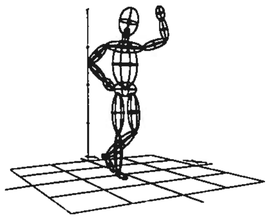
CHAPTER THREE



Nested Transformations and Blobby Man

OCTOBER 1987

There are a lot of interesting things you can do with transformation matrices. Later chapters will deal with this quite a bit, so I will spend some time here describing my notational scheme for nested transformations. As a non-trivial example I will include the database for an articulated human figure called *Blobby Man*. (Those of you who already know how to do human articulation, don't go away. There are some cute tricks here that are very useful.)

The Mechanism

This is an implementation of the well-known technique of nested transformations. (Don't you just hate it when people call something "well known" and you have never heard of it? It sounds like they are showing off how many things they know. Well, admittedly we can't derive *everything* from scratch. But is sure would be nice to find a less smug way of saying so.)

For those for whom this is not so well known, the basic idea behind nested transformations appears in several places, notably in Foley and van Dam[1] and in Glassner.[2] It is just an organizational scheme to make it easier to deal with a hierarchy of accumulated transformations. It shows up in various software systems and has hardware implementations in the E&S Picture System or the Silicon Graphics IRIS.

1 James D. Foley and Andries van Dam, *Fundamentals of Interactive Computer Graphics* (Reading, Mass.:

Instructions for rendering a scene take the form of a list of commands and their parameters. These will be written here in TYPEWRITER type. All commands will have four or fewer letters. (The number 4 is used because of its ancient numerological significance.) Parameters will be separated by commas, not blanks. (Old-time FORTRAN programmers don't even see blanks, let alone use them as delimiters.) Don't complain, just be glad I'm not using O-Language (maybe I'll tell you about *that* sometime).

Basic Command Set

These commands modify **C** and pass primitives through it. Each modification command premultiplies some simple matrix into **C**. No other action is taken. The command descriptions below will explicitly show the matrices used.

Translation

```
TRAN x, y, z
```

premultiplies **C** by an elementary translation matrix.

$$\mathbf{C} \leftarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ x & y & z & 1 \end{bmatrix} \mathbf{C}$$

Scaling

```
SCAL sx, sy, sz
```

premultiplies **C** by an elementary scaling matrix.

$$\mathbf{C} \leftarrow \begin{bmatrix} sx & 0 & 0 & 0 \\ 0 & sy & 0 & 0 \\ 0 & 0 & sz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{C}$$

Rotation

```
ROT θ, j
```

Briefly, it works like this. We maintain a global 4 × 4 homogeneous coordinate transformation matrix called the *current transfomation*, **C**, containing the transformation from a primitive's *definition space* onto a desired location in *screen space*. I will assume a *device-independent* (buzz, buzz) screen space ranging from −1 to +1 in x and y and where z goes *into* the screen. This is a *left-handed* coordinate system.

Each time a primitive is drawn, it is implicitly transformed by **C**. For example, the transformation of a (homogeneous) point is accomplished through simple matrix multiplication.

$$[x, y, z, w]_{scrn} = [x, y, z, w]_{defn}\mathbf{C}$$

Other primitives can be transformed by some more complex arithmetic involving this matrix.

C is typically the product of a perspective transformation and various rotations, translations, and scales. It is built up with a series of matrix multiplications by simpler matrices. Each multiplication *premultiplies* a new matrix into **C**.

$$\mathbf{C} \leftarrow \mathbf{T}_{new}\mathbf{C}$$

Why in this order? Because a collection of objects, subobjects, subsubobjects, etc., is thought of as a tree-like structure. Drawing a picture of the scene is a top-down traversal of this tree. You encounter the more global of the transformations first and must multiply them in as you see them. The transformations will therefore seem to be applied to the primitives in the *reverse* order to that in which they were multiplied into **C**. Another way you can think of it is that the transformations are applied in the *same* order stated, but that the *coordinate system* transforms along with the primitive as each elementary transformation is multiplied. At each node in the tree, of course, you can save and restore the current contents of **C** on a stack.

The Language

The notational scheme I will use is not just a theoretical construct, it's what I actually use to do all my animations. It admittedly has a few quirks, but I'm not going to try to sanitize them because I want to be able to use databases I have actually tried out and to show listings that I know will work. I have purposely made each operation *very* elementary to make it easy to experiment with various combinations of transformations. Most reasonable graphics systems use something like this, so it shouldn't be too hard for you to translate my examples into your own language.

The depth transformation is specified by two values—z_n (the location of the *near* clipping plane) and z_f (the location of the *far* clipping plane). The matrix transforms z_n to +0, and z_f to +1. I know that the traditional names for these planes are *hither* and *yon*, but for some reason I always get these words mixed up, so I call them *near* and *far*.

Precalculate the following quantities (note that far clipping can be effectively disabled by setting $z_f = \infty$, which makes $Q = s$).

$$s = \sin(\alpha/2)$$

$$c = \cos(\alpha/2)$$

$$Q = \frac{s}{1 - z_n/z_f}$$

The matrix is then

$$\mathbf{C} \leftarrow \begin{bmatrix} c & 0 & 0 & 0 \\ 0 & c & 0 & 0 \\ 0 & 0 & Q & s \\ 0 & 0 & -Qz_n & 0 \end{bmatrix} \mathbf{C}$$

Orientation

```
ORIE a, b, c, d, e, f, p, q, r
```

Sometimes it's useful to specify the rotation (orientation) portion of the transformation explicitly. There is nothing, though, to enforce it being a pure rotation, so it can be used for skew transformations.

$$\mathbf{C} \leftarrow \begin{bmatrix} a & d & p & 0 \\ b & e & q & 0 \\ c & f & r & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{C}$$

Transformation Stack

```
PUSH
POP
```

These two commands push and pop **C** on/off the stack.

The j parameter is an integer from 1 to 3 specifying the coordinate axis (x, y, or z). The positive rotation direction is given via the Right-Hand Rule (if you are using a left-handed coordinate system) or the Left-Hand Rule (if you are using a right-handed coordinate system). This may sound strange, but it's how it's given in Newman and Sproull.[3] It makes *positive* rotation go *clockwise* when viewing in the direction of a coordinate axis. For each matrix below, we precalculate

$$s = \sin\theta$$
$$c = \cos\theta$$

The matrices are then

$j = 1$ (x axis)

$$\mathbf{C} \leftarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c & -s & 0 \\ 0 & s & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{C}$$

$j = 2$ (y axis)

$$\mathbf{C} \leftarrow \begin{bmatrix} c & 0 & s & 0 \\ 0 & 1 & 0 & 0 \\ -s & 0 & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{C}$$

$j = 3$ (z axis)

$$\mathbf{C} \leftarrow \begin{bmatrix} c & -s & 0 & 0 \\ s & c & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{C}$$

Perspective

PERS α, z_n, z_f

This transformation combines a perspective distortion with a depth (z) transformation. The perspective assumes the eye is at the origin, looking down the +z axis. The field of view is given by the angle α.

cueing, not depth *queueing* as I've seen some people write. (Depth queueing *could* perhaps be used to refer to a depth-priority rendering algorithm . . . hmmm.)

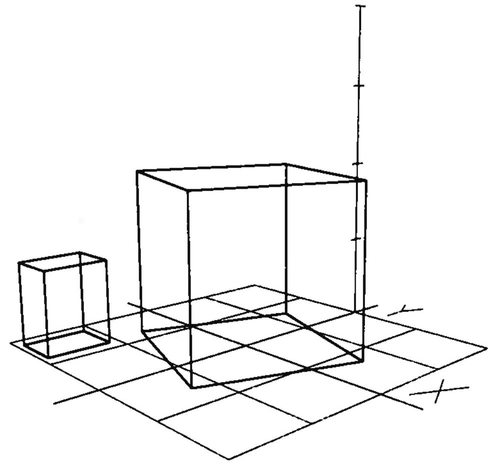


Figure 3.1 *Cubes on parade*

Possible Implementations

There are several ways you could perform the operations described by these lists of commands.

- Translate them into explicit subroutine calls in some language implementation and compile them.
- Read them through a "filter"-type program that executes the commands as they are encountered. This is the way most of my rendering programs work.
- Read them into an "editor"-type program that tokenizes the commands into some interpreter data structure and reexecutes the sequence upon each frame update. This is the way my animation design program works.

Advanced Commands

The simple commands above can be implemented in about two pages of code. The enhancements below are a little more elaborate. The following constructions make sense only in the "editor" mode of operation.

Parameters

Any numeric parameter can be given a symbolic name. A symbol table will be maintained and the current numeric value of the symbol used when the instruction is executed. For example, our cube scene could be

```
PERS FOV, ZN, ZF
TRAN XSCR, YSCR, ZSCR
ROT BACK, 1
ROT SPIN, 3
SCAL 1, 1, -1
DRAW GPLANE
PUSH
```

Primitives

```
DRAW name
```

A primitive could be a list of vector endpoints, points-and-polygons, implicit surfaces, cubic patches, blobbies, etc. This command means "pass the elements in primitive *name* (however it's defined) through **C** and onto the screen."

Example

A typical scene will consist of an alternating sequence of **C**-alteration commands and of primitive-drawing commands. At the beginning of the command list, **C** is assumed to be initialized to the identity matrix.

Here is a typical sequence of commands to draw a view of two cubes sitting on a grid plane. The primitive GPLANE consists of a grid of lines in the *xy* plane covering –2 to +2 along each axis, along with some labels and a tick-marked pole in the +*z* direction that is placed at $y = 2$. The primitive CUBE consists of a cube whose vertices have coordinates $[\pm 1, \pm 1, \pm 1]$—that is, it is centered at the origin and has edge length equal to 2. Notice the scale by –1 in *z* to convert from the right-handed system in which the scene is defined to the left-handed system in which it is rendered.

```
PERS 45, 6.2, 11.8
TRAN 0, -1.41, 9
ROT -80, 1
ROT 48, 3
SCAL 1, 1, -1
DRAW GPLANE
PUSH
TRAN 0, 0, 1
ROT 20,3
DRAW CUBE
POP
PUSH
SCAL .3, .4, .5
TRAN -5, -3.8, 1
DRAW CUBE
POP
```

The results of executing these instructions appear in Figure 3.1.

Notice that the z_n and z_f variables are selected to bound the scene as closely as possible so that depth cueing will work. And hey, it's called depth

```
ROT SPIN, 3
SCAL 1, 1, -1
DRAW GPLANE
DRAW CUBE, TRAN,X1,Y1,Z1, ROT,ANG,3
DRAW CUBE, SCAL,.3,.4,.5, TRAN,-5,-3.8,Z1
```

Subassembly Definitions

These are essentially subroutines. A subassembly is declared and named by bracketing its contents by the commands

```
DEF name
  :
any commands
  :
----
```

Once defined, a subassembly can be thought of as just another primitive. In fact, the "designer" of a list of commands should not know or care if the thing they are drawing is a primitive or a subassembly, so a subassembly is "called" by the same command as a primitive.

```
DRAW assy_name
```

The subassembly calling and return process is completely independent of the matrix stack PUSH and POP process. Interpretation of commands begins at the built-in name WORLD.

I typically organize my definitions so that WORLD contains only the *viewing transformation*, i.e., its rotations and transformations tell where the "camera" is and in which direction it is looking. My favorite all-purpose viewing transform is

```
DEF WORLD
PERS FOV, ZN, ZF
TRAN XSCR, YSCR, ZSCR
ROT BACK, 1
ROT SPIN, 3
ROT TILT, 1
TRAN -XLOOK, -YLOOK, -ZLOOK
SCAL 1, 1, -1
DRAW SCENE
----
```

The variables XLOOK, YLOOK, and ZLOOK determine the "look-at" point.

```
ROT ANG, 3
DRAW CUBE
POP
PUSH
SCAL .3, .4, .5
TRAN -5, -3.8, Z1'
DRAW CUBE
POP
```

By setting the variables

```
FOV = 45     ZN = 6.2       ZF = 11.8
XSCR = 0     YSCR = -1.41   ZSCR = 9
BACK = -80   SPIN = 48
X1 = 0       Y1 = 0         Z1 = 1
ANG = 20
```

and executing the command list, the same results would be generated. The same symbol can appear in more than one place, allowing a certain amount of constraint satisfaction.

Abbreviations

Each time a subobject is positioned relative to a containing object, the instructions usually look something like

```
PUSH
  :
  :
various TRAN, ROT, SCAL commands
  :
  :
DRAW primitive
POP
```

While explicit, the above notation is sometimes a bit spread out and hard to follow. This sort of thing happens so often that it's helpful to define an abbreviation for it. We do so by following the DRAW command (on the same line) by the list of transformation commands, separated by commas. An implied PUSH and POP encloses the transformation list and DRAW. Our cube scene now looks like

```
PERS FOV, ZN, ZF
TRAN XSCR, YSCR, ZSCR
ROT BACK, 1
```

Table 3.1 *Meanings of Blobby Man variables*

EXTEN	Extension. A dancers' term for bending forwards and backwards (*x* axis)
ROT	Rotation. A dancers' term for rotating the body and shoulders left and right about the vertical (*z*) axis
BTWIS	Angle of body leaning left and right (*y* axis)
NOD	Head nod
NECK	Head shake
LHIP, RHIP	Angular direction that the leg is kicked
LOUT, ROUT	Angular distance that the leg is kicked
LTWIS, RTWIS	Angle the leg is twisted about its length
LKNEE, RKNEE	Knee bend
LANKL, RANKL	Ankle bend
LSID, RSID	Arm rotation to side
LSHOU, RSHOU	Arm rotation forwards and back
LATWIS, RATWIS	Arm rotation about its own length
LELBO, RELBO	Elbow angle

```
DEF BODY
DRAW SPHERE, TRAN,0,0,0.62, SCAL,0.306,0.21,0.5,
DRAW SHOULDER, TRAN,0,0,1, ROT,EXTEN,1, ROT,BTWIS,2, ROT,ROT,3,
----

DEF SHOULDER
DRAW SPHERE, SCAL,0.45,0.153,0.12,
DRAW HEAD, TRAN,0,0,0.153, ROT,NOD,1, ROT,NECK,3,
DRAW LEFTARM, TRAN,-0.45,0,0, ROT,LSID,2, ROT,LSHOU,1, ROT,LATWIS,3,
DRAW RGHTARM, TRAN, 0.45,0,0, ROT,RSID,2, ROT,RSHOU,1, ROT,RATWIS,3,
----
```

```
DEF LEFTLEG
PUSH
ROT LHIP, 3,
ROT LOUT, 2,
ROT -LHIP, 3,
```

```
DEF RGHTLEG
PUSH
ROT RHIP, 3,
ROT ROUT, 2,
ROT -RHIP, 3,
```

YSCR, and ZSCR position the "look-at" point on the screen. XSCR and YSCR might very well be zero, but ZSCR needs to be some positive distance to move the scene away from the eye.

The assembly SCENE contains the contents of the scene and can be designed independently of how it is being viewed. Our cube scene again:

```
DEF SCENE
DRAW GPLANE
DRAW CUBE, TRAN,X1,Y1,Z1, ROT,ANG,3
DRAW CUBE, SCAL,.3,.4,.5, TRAN,-5,-3.8,Z1
----
```

Blobby Man

A few years ago I made a short animation of a human figure called Blobby Man to illustrate a new surface modeling technique.[4] Leaving aside issues of modeling, the figure itself is an interesting example of nested transformations. I have, in fact, used it as a homework assignment for my computer graphics class. (Gee, I guess I can't do that any more.)

Blobby Man's origin is in his stomach, and he stands with the *z* axis vertical. The only primitive element is a unit radius SPHERE centered at the origin. The parameterized variables are all rotation angles. Their usage is defined in Table 3.1.

The WORLD is the standard one given above. SCENE looks like

```
DEF SCENE
DRAW GPLANE
DRAW TORSO, TRAN,XM,YM,ZM, ROT,RZM,3,
----
```

The actual articulated parts are

```
DEF TORSO
DRAW LEFTLEG, TRAN,-0.178,0,0,
DRAW RGHTLEG, TRAN,0.178,0,0,
DRAW SPHERE, TRAN,0,0,0.08, SCAL,0.275,0.152,0.153,
DRAW BODY, ROT,EXTEN,1, ROT,BTWIS,2, ROT,ROT,3,
----
```

```
DEF THIGH
DRAW SPHERE, TRAN,0,0,-0.425, SCAL,0.141,0.141,0.425,
----

DEF CALF
DRAW SPHERE, SCAL,0.05,0.05,0.05,
DRAW SPHERE, TRAN,0,0,-0.425, SCAL,0.1,0.1,0.425,
----

DEF FOOT
DRAW SPHERE, SCAL,0.05,0.04,0.04,
DRAW SPHERE, TRAN,0,0.05,-0.05, SCAL,0.04,0.04,0.04,
DRAW SPHERE, TRAN,0,-0.15,-0.05, ROT,-10,1, SCAL,0.08,0.19,0.05,
----
```

A picture of the result appears in Figure 3.2. The viewing parameters are

```
ZN = 5.17     ZF = 10.7
XSCR = -.1    YSCR = -1.6    ZSCR = 7.9
BACK = -90    SPIN = -30     TILT = 0
XLOOK = 0     YLOOK = 0      ZLOOK = 0
XM = 0        YM = 0         ZM = 1.75
```

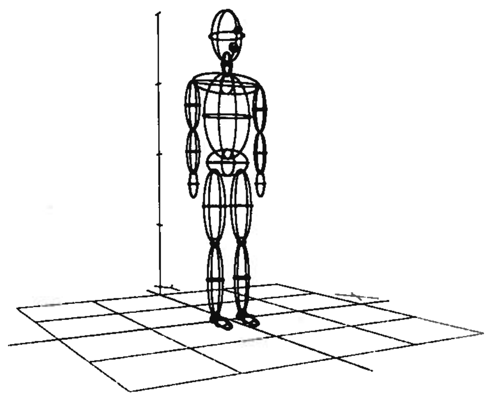


Figure 3.2 *Blobby Man*

All other angles are 0.

A picture of the man gesturing is in Figure 3.3. The view is the same, but the body angles are

```
NOD = -25     NECK = 28
RHIP = 105    ROUT = 13     RTWIS = -86      RKNEE = -53
LHIP = 0      LOUT = 0      LTWIS = 0        LKNEE = 0
LSID = -45    LSHOU = 0     LATWIS = -90     LELBO = 90
RSID = 112    RSHOU = 40    RATWIS = -102    RELBO = 85
```

There are several tricks in the model of Blobby Man that are especially notable.

Cumulative Transformations

It is not necessary to POP a transformation just after it is used to DRAW something. Sometimes it is useful to continuously accumulate translations and rotations. For example, Blobby Man's leg could have looked like

```
DRAW THIGH              DRAW THIGH
TRAN 0, 0, -0.85,       TRAN 0, 0, -0.85,
ROT LKNEE, 1,           ROT RKNEE, 1,
DRAW CALF               DRAW CALF
TRAN 0, 0, -0.84,       TRAN 0, 0, -0.84,
ROT LANKL, 1            ROT RANKL, 1
DRAW FOOT               DRAW FOOT
POP                     POP
----                    ----

DEF LEFTARM             DEF RGHTARM
PUSH                    PUSH
DRAW UPARM              DRAW UPARM
TRAN 0, 0, -0.55,       TRAN 0, 0, -0.55,
ROT LELBO, 1,           ROT RELBO, 1,
DRAW LOWARM             DRAW LOWARM
TRAN 0, 0, -0.5,        TRAN 0, 0, -0.5,
DRAW HAND               DRAW HAND
POP                     POP
----                    ----
```

Some primitive body parts are defined as translated and squashed spheres as follows:

```
DEF HEAD
DRAW SPHERE, TRAN,0,0,0.4, SCAL,0.2,0.23,0.3
DRAW SPHERE, TRAN,0,-0.255,0.42, SCAL,0.035,0.075,0.035,
DRAW SPHERE, TRAN,0,0,0.07, SCAL,0.065,0.065,0.14
DRAW SPHERE, TRAN,0,-.162,.239, SCAL,.0533,.0508,.0506,
----


DEF UPARM
DRAW SPHERE, TRAN,0,0,-0.275, SCAL,0.09,0.09,0.275,
----

DEF LOWARM
DRAW SPHERE, TRAN,0,0,-0.25, SCAL,0.08,0.08,0.25,
----

DEF HAND
DRAW SPHERE, TRAN,0,0,-0.116, SCAL,0.052,0.091,0.155,
----
```

wanted to rotate a primitive about a point at coordinates (DX, DY), the commands would be

```
TRAN DX, DY, 0
ROT ANGLE, 3
TRAN -DX, -DY, 0
```

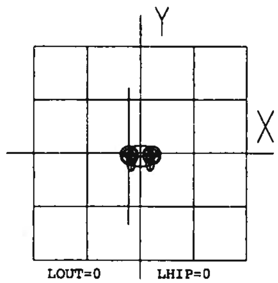
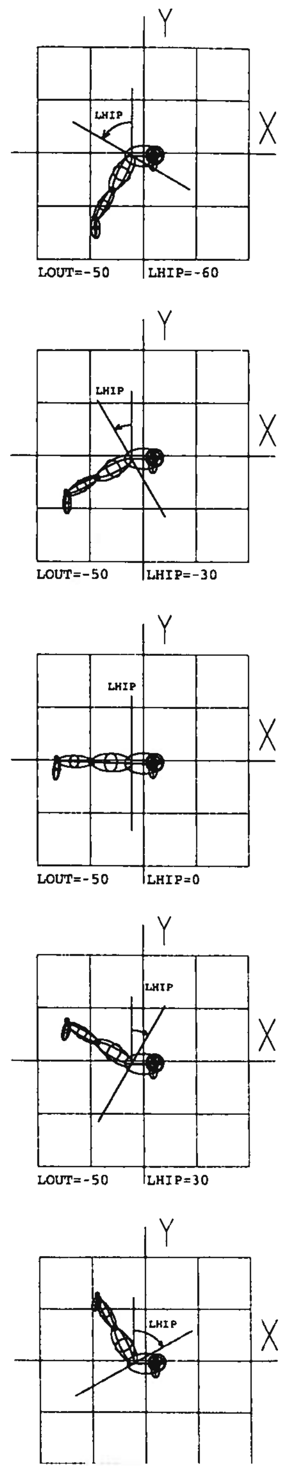


Figure 3.4 *Top view of leg rotation*



In other words, you translate the desired rotation center to the origin, rotate, and then translate the center back to where it used to be. (Remember that the transformations will be effectively carried out in sequence in the *reverse* order from that seen above.) The rotation sequence used for the leg enables us to rotate the leg about a rotated coordinate axis. The purpose of this is to make the foot always point forwards, no matter what LHIP and LOUT are. Figure 3.4 shows how this works. It is a top view of just the legs and hips, and the dark line shows the axis of rotation by the angle LOUT. A similar technique could have been used for the arm-shoulder joints, but I didn't happen to need that much flexibility in the animation.

Addendum

I received a letter from Nelson Max about this chapter. He pointed out that the rotation trick for making the foot always point forwards does not keep it *exactly* forwards (with an x component of 0). It still has some small sideways component. This is, of course, quite true. My intention was just to keep it approximately pointing forwards (with a negative y component). This works best for the expected range of values $-90 <$ LOUT < 0 and $-90 <$ LHIP < 90. All other rotation combinations I tried made it too easy to get the foot pointing completely backwards, amusing perhaps, but a real nuisance for animation.

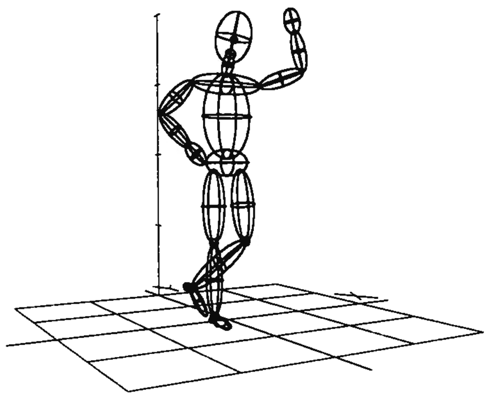


Figure 3.3 *Blobby Man waving*

```
DEF LLEG
DRAW THIGH
DRAW CALFETC, TRAN,0,0,-0.85,
ROT,LKNEE,1
----

DEF CALFETC
DRAW CALF
DRAW FOOT, TRAN,0,0,-0.84, ROT,LANKL,1
----
```

As long as there are no transformed objects after the last one, some of the nesting can be dispensed with, leaving . . .

```
DEF LLEG
PUSH
DRAW THIGH
TRAN 0, 0, -0.85
ROT LKNEE, 1
DRAW CALF
TRAN 0, 0, -0.84
ROT LANKL, 1
DRAW FOOT
POP
----
```

Repeated Variables

The variables EXTEN, BTWIS, and ROT are used twice, once to flex the BODY relative to the TORSO and once to flex the SHOULDER relative to the BODY. This gives a minimal simulation of a flexible spine for the figure.

Rotated Rotations

The transformation of the (left) leg relative to the torso contains the sequence

```
ROT LHIP, 3
ROT LOUT, 2
ROT -LHIP, 3
```

This is something I'm especially proud of. It is a not-completely-obvious