

Math 597 Project Summary: Tubulating the Torus Knot

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1 Theory

The parametrization for a torus is:

$$\begin{aligned}x &= (R + r\cos(\theta))\cos(\phi) \\y &= (R + r\cos(\theta))\sin(\phi) \\z &= r\sin(\theta)\end{aligned}$$

The parametrization for a (p,q) torus knot lying on this torus is:

$$\begin{aligned}x &= (R + r\cos(pt))\cos(qt) \\y &= (R + r\cos(pt))\sin(qt) \\z &= -r\sin(pt)\end{aligned}$$

where $t \in [0, 2\pi]$.

To tube the knot, we use FrenetSerret frame (TNB frame).

$$T = \frac{v}{|v|} \text{the unit vector tangent to the curve.} \quad (1)$$

$$N = \frac{\dot{v} \times v}{|\dot{v} \times v|} \text{the normal vector} \quad (2)$$

$$B = \frac{v \times N}{|v \times N|} \quad (3)$$

So the points of the tube are of the form:

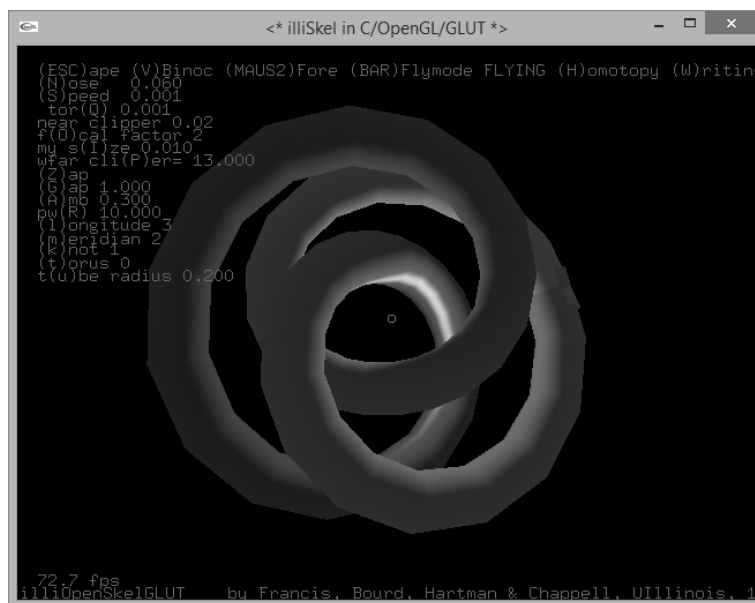
$$\vec{r} + a\cos(\tau)N + a\sin(\tau)B$$

where a is the radius of the tube and $\tau \in [0, 2\pi]$.

2 OpenGL Realization

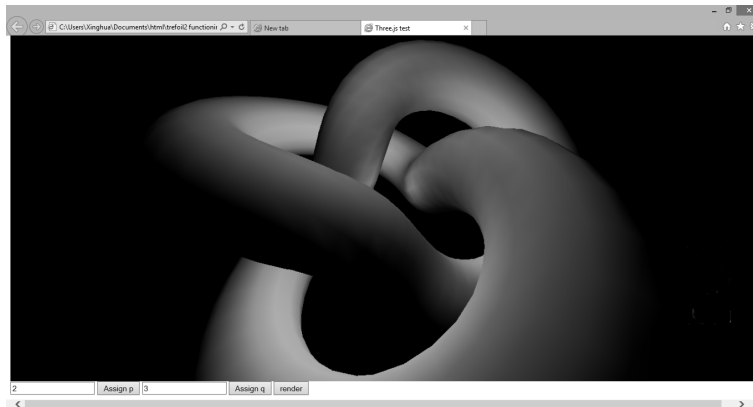
The program pc0GF.c is based on skel.c. Here are some instruction on how to use the program:

- Press t to show the original torus. Press k to show the trefoil.
- Press m/M and l/L to increase/decrease p(meridian count) and q(longitude count).
- left/right click to rotate counterclockwise/clockwise.
- press space to enter/quit flying mode. Use middle button to fly.
- press s to change the speed of flying.
- press u to change the radius of the tube.
- press G to create gap on the torus and knot.
- press Z to reset.



3 Javascript 3D realization

The "trefoil2.html" use the threejs gemetry called TorusKnotGeometry. Enter p q value and assign. Click render to show the knot. The original p and q are both 1.



The "figure8.html" uses the threejs geometry called TubeGeoentry. You can tube any curve with this library once you know the parametrization of the curve. I tube the figure eight knot in this example. Use a w s d key to move the object. Use mouse to rotate.

