

## 1 Abstract

The optimization of materials is a key part of materials science and engineering. Interestingly, the perfecting process often takes advantage of the natural problems present in materials, entitled defects. Typically, these defects are classified according Euclidean dimensions (0, 1, 2, etc.).<sup>1</sup> While this approach works for certain defects, such as those in Table 1.1: Euclidean Defects, it does not work for defects that are curvilinear or those that cannot be described using standard equations.

The other defects, since they do not exist in any Euclidean (integer) dimension, must lie in fractional (fractal) dimension, denoted by  $D$  (see Table 1.2: Non-Euclidean Defects). One such defect, the grain boundary, whose dimension is between one and two, can be used to measure the fracture toughness of the material.<sup>2</sup> As such, production processes that vary the fractal dimension of the grain boundary can be used to strengthen or weaken a material.

According to studies by H. Khanbareh, J. H. Kruhl, and M. Nega<sup>3</sup>, grain boundaries can be approximated by the Koch Curve fractal. The traditional Koch curve, which starts with a one-dimensional line segment and exists in the dimension  $\log 4 / \log 3$ , repeats according to the following process:

1. Take an equilateral triangle.
2. Split the sides into three equal lengths.
3. On any outward facing side (a side that is not connected to a previous triangle), take the central length and create an equilateral triangle from it.
4. For each line segment in the set, repeat from 1.<sup>4</sup>

Other versions of the Koch curve manipulate the initial line segment in different manners. Technically, the curve will remain a Koch curve fractal if it is self-similar and exists in a fractional dimension between one and two. With some extrapolation, the Koch process can be extended to higher dimensional starting points. In particular, if the starting point is a three-dimensional object, such as a regular tetrahedron or a cube (analogs of the triangular and square patterns for  $D$  between 1 and 2), then the resulting form will be a fractal with  $D$  between 3 and 4.

<sup>1</sup>Callister, William D. & Rethwisch, David G. (2012) Hoboken, NJ: John Wiley & Sons, Inc.

<sup>2</sup>Khanbareh. H. (2011, December). Fractal Dimension Analysis of Grain Boundaries of 7XXX Aluminum Alloys and Its Relationship to Fracture Toughness. Retrieved from [http://www.lr.tudelft.nl/fileadmin/FaculteitLR/Images/NovAMPictures\\_researchMSc\\_projects/Hamide\\_thesis.pdf](http://www.lr.tudelft.nl/fileadmin/FaculteitLR/Images/NovAMPictures_researchMSc_projects/Hamide_thesis.pdf)

<sup>3</sup>See 2, Kruhl, J.H. & Nega, M. (1996). Geologische Rundschau, 85, 38-43. DOI:10.1007/BF00192058

<sup>4</sup>Weisstein, Eric W. (2013). <http://mathworld.wolfram.com/KochSnowflake.html>

Table 1.1: Euclidean Defects

Dimension (d)	Type of Defect
0	Point defect
1	Linear defect
2	Planar defect
3	3-dimensional defect

Table 1.2: Non-Euclidean Defects

Dimension (D)	Type of Defect
(1,2)	Grain boundary
(2,3)	Surface defect

## 2 Accomplishments

Over the past week, I have accomplished the following:

1. Received acceptance for my proposal.
2. Changed my programming language from VPython to Python + OpenGL (Py-OGL), with the approval of Professor Francis.
3. Worked on a micro-project in Py-OGL which consisted of altering the example octahedron project `oc1.py`, available on the Math 198 website. Created triangles by connecting the midpoints of each side of the octahedron. Placed my program, titled `oc2.py`, in the `semibra2/pyopenGL` folder in the repository.
4. Created documentation for the `oc2.py` program and placed it in the `semibra2/pyopenGL` folder in the repository.
5. Researched common commands in vim, the shell text editing program.
6. Worked towards creating pseudo-code for the tetrahedral Koch "star."
7. Updated my webpage.
8. Changed the timeline specified in my Proposal. Point 2 will be accomplished by November 1st, F10. Points 4 and 5 will be accomplished by November 15th, F12.