Higher-Dimensional Koch Curves

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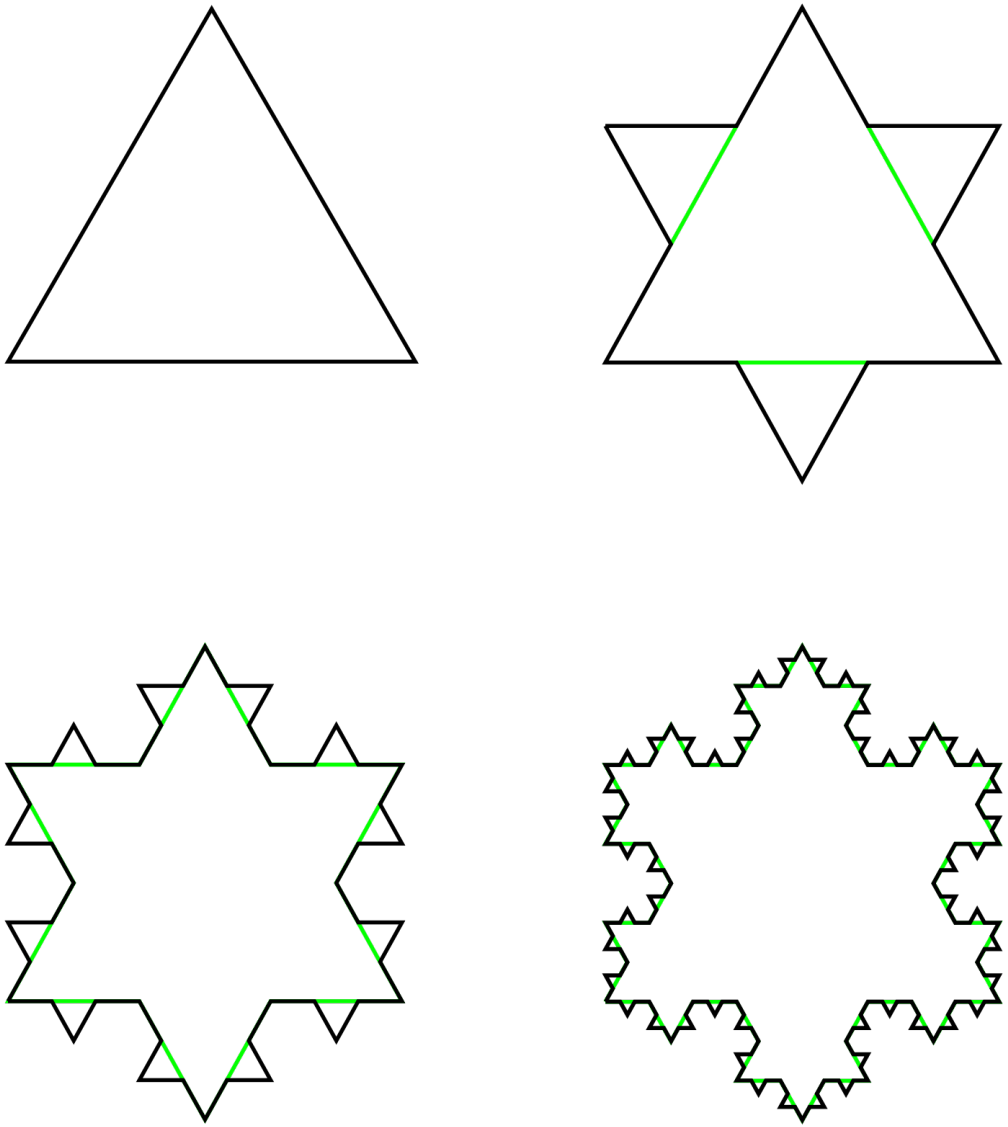
Koch Curve



URL: http://davidprice.files.wordpress.com/2013/07/2000px-kochflake.png

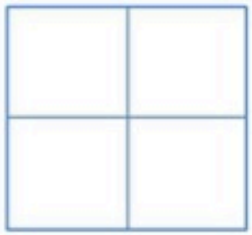
Dimension

Topological Dimension	Object
0	Point
1	Line
2	Plane
3	Space

In between?

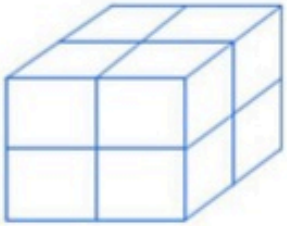
1-D N parts, scaled by ratio r = $1/N$

$$N r^1 = 1$$



2-D N parts, scaled by ratio r = $1/N^{1/2}$

$$N r^2 = 1$$



3-D N parts, scaled by ratio r = $1/N^{1/3}$

$$N r^3 = 1$$

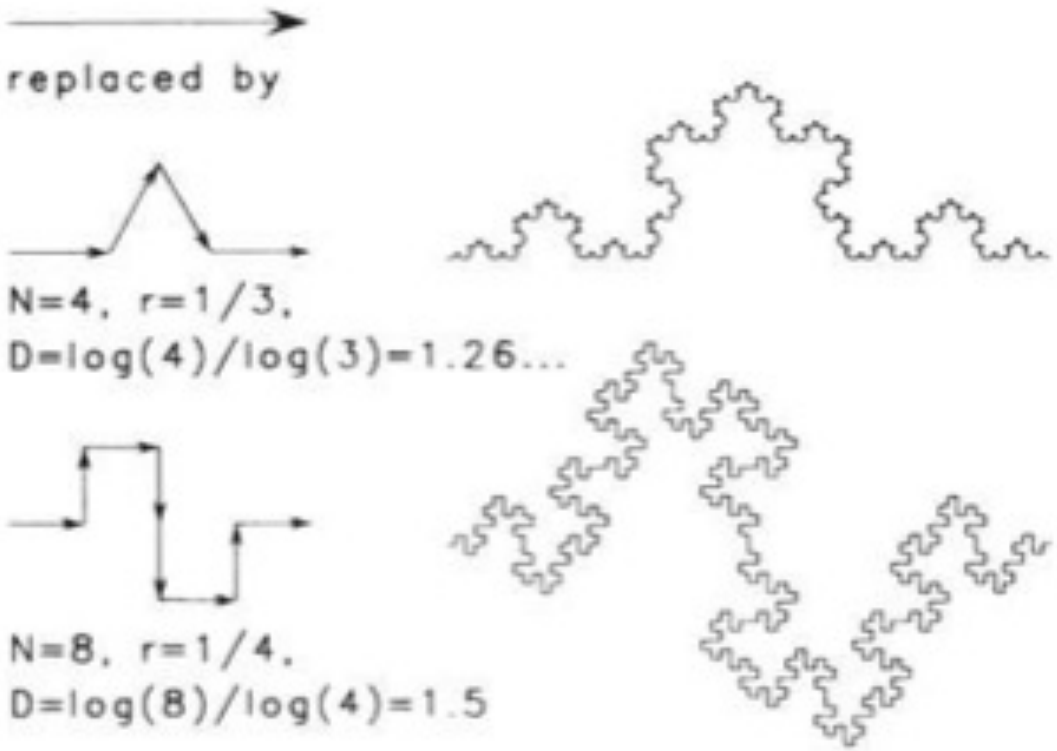
Generalize

For an object of N parts, each scaled down by a ratio r from the whole:

$$N r^D = 1$$

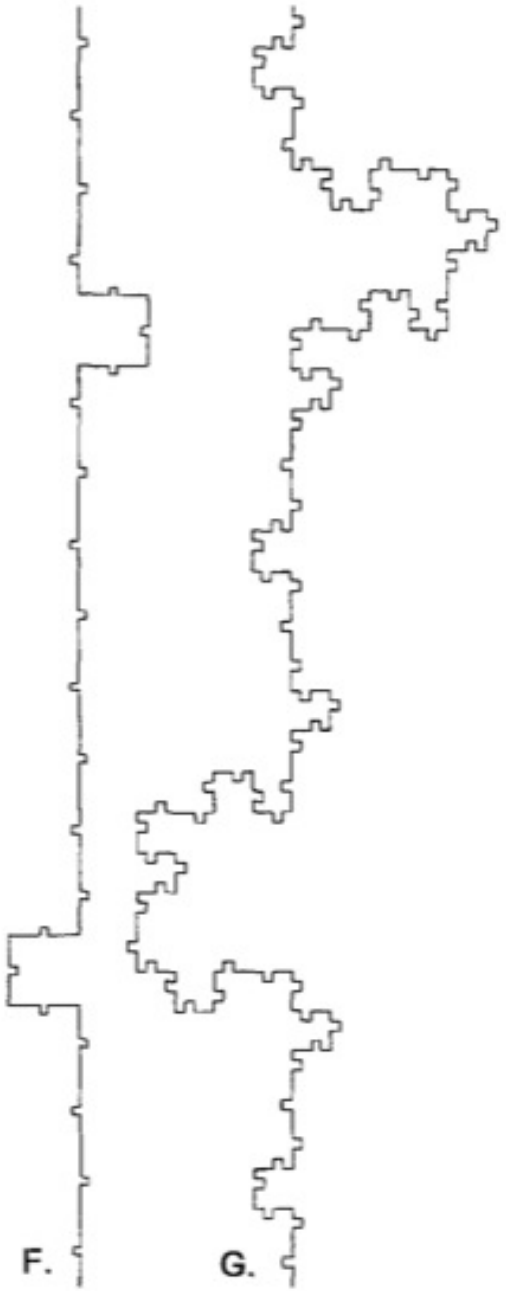
$$D = \frac{\log N}{\log 1/r}$$

Generating Different Dimensions



Curve F:
D≈1.051

Curve G:
D≈1.293



Defects in Materials

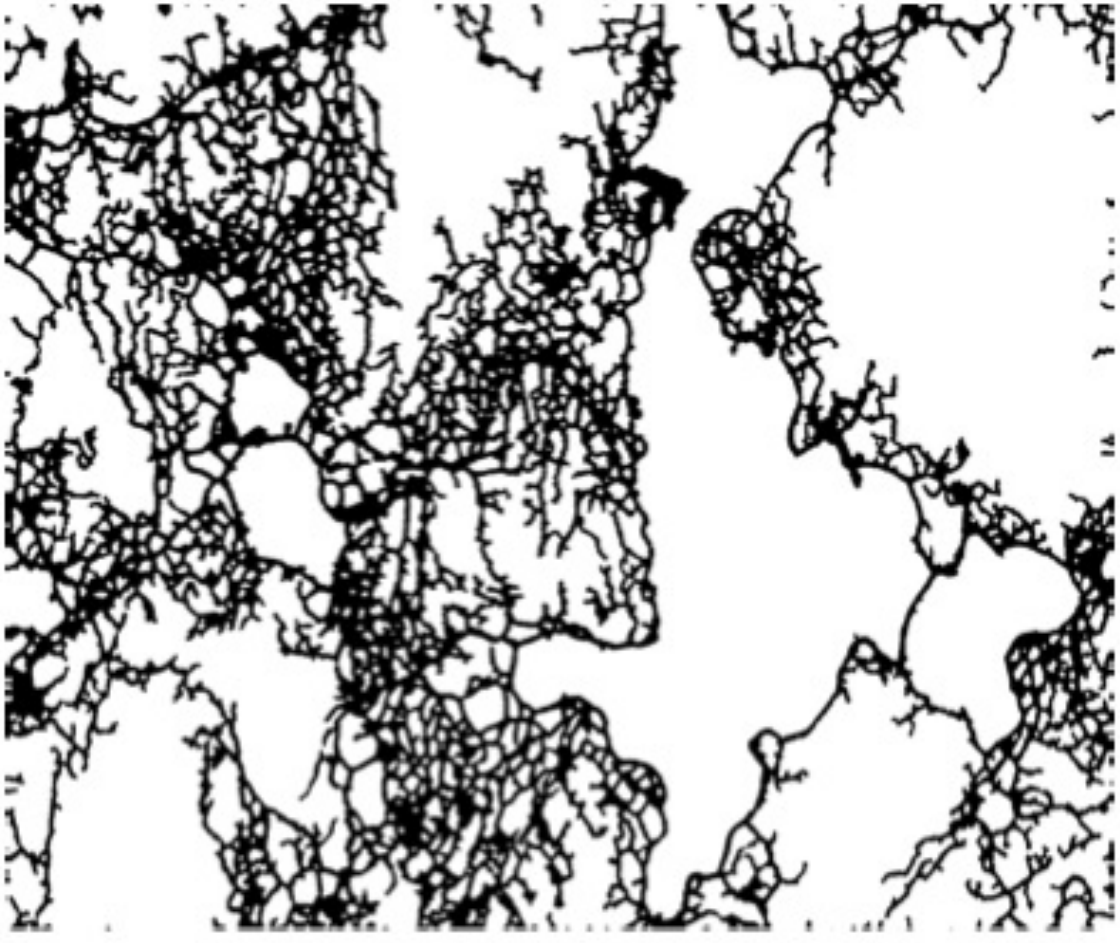
Dimension (d)	Defect
0	Point defect • Interstitial • Substitutional • Vacancy
1	Line defect • Dislocation
2	Plane defect • Twin planes (twinning)
3	Three dimensional defect

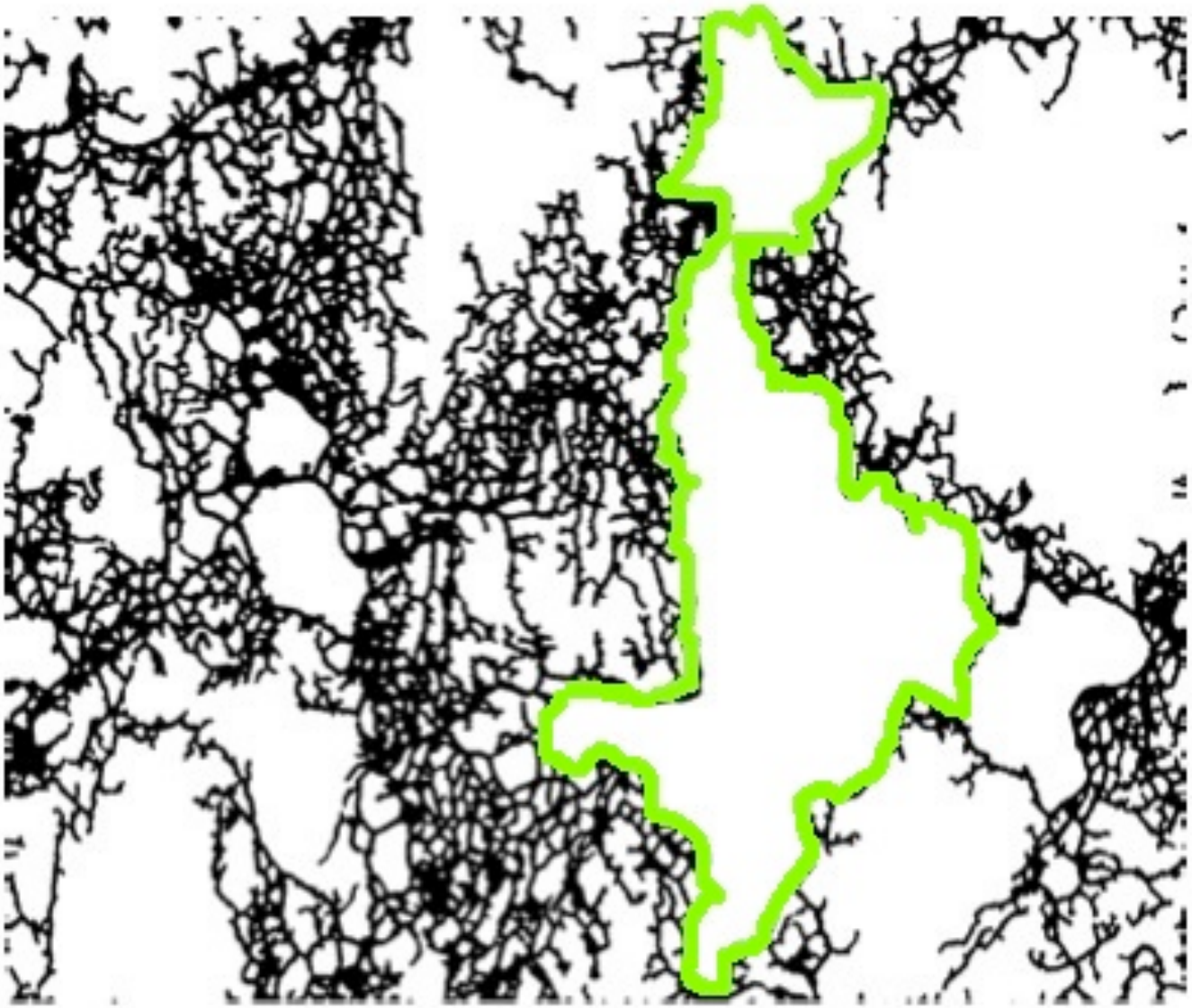
Fract(ion)al Dimension

Dimension (D)	Defect
(1,2)	Grain boundary
(2,3)	Surface defect

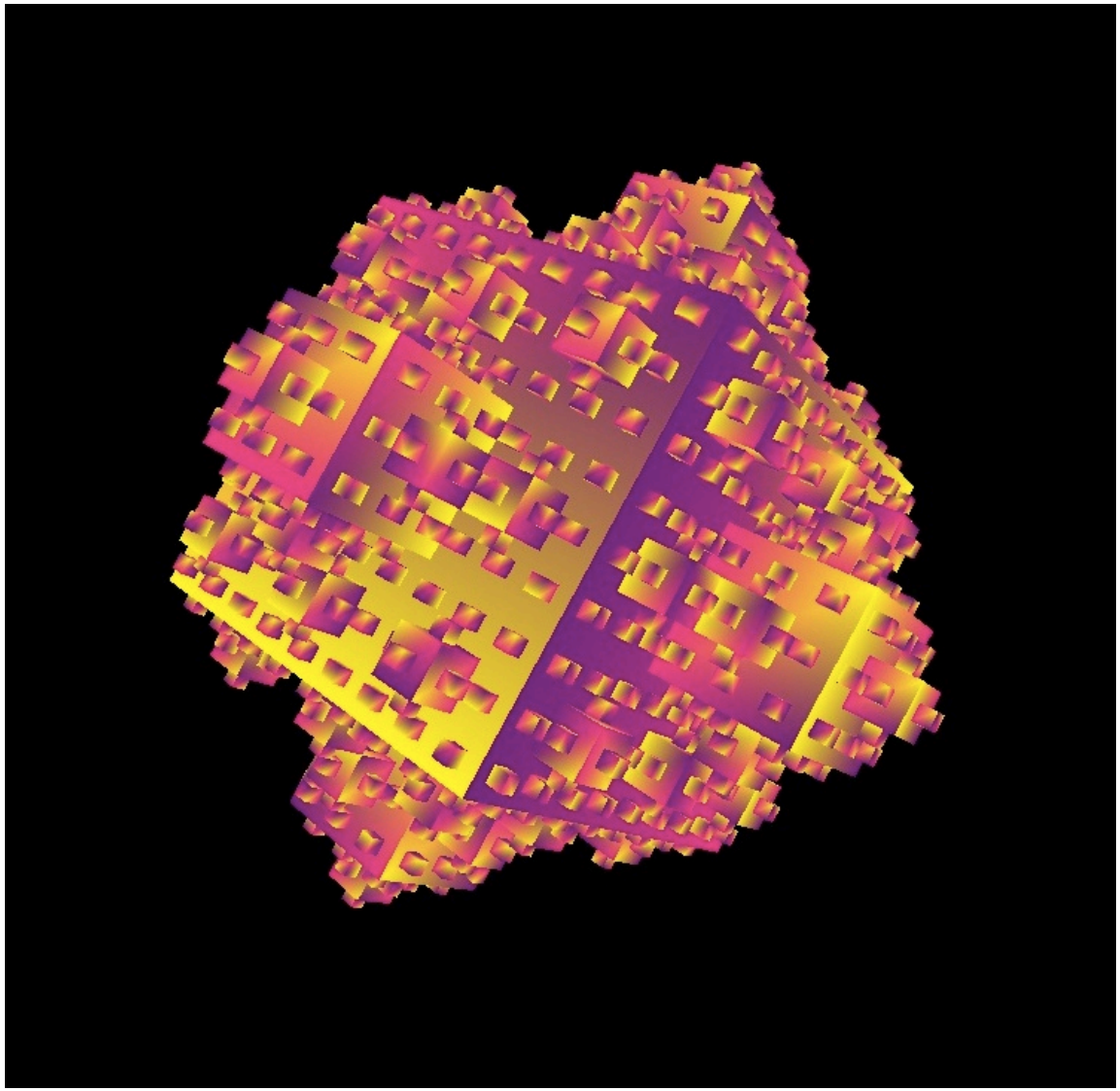
Grain Boundary

- Grain boundary in Al7050

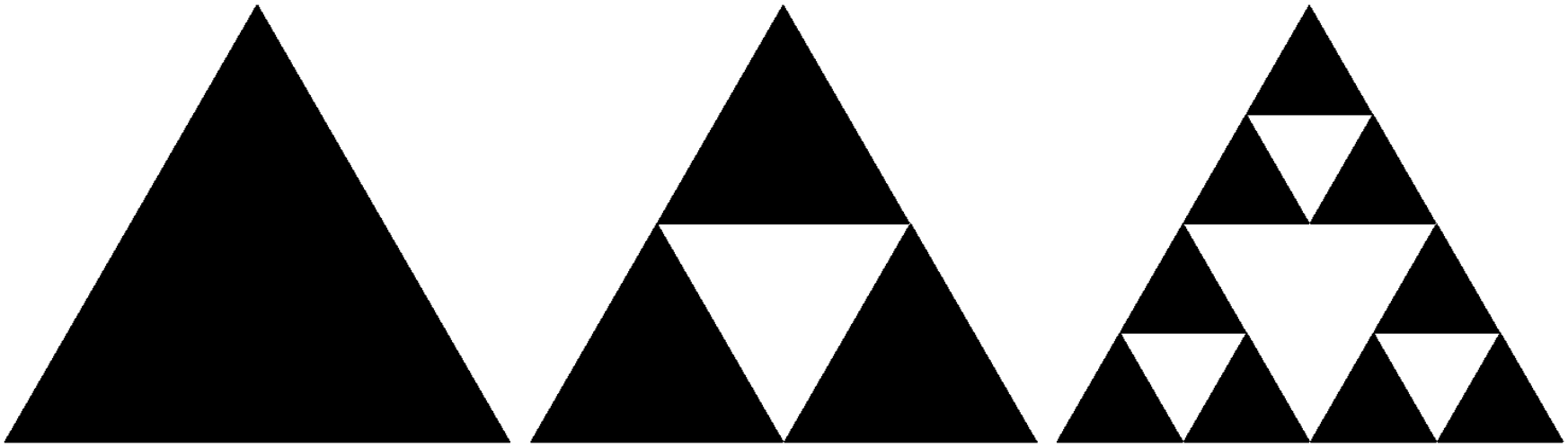




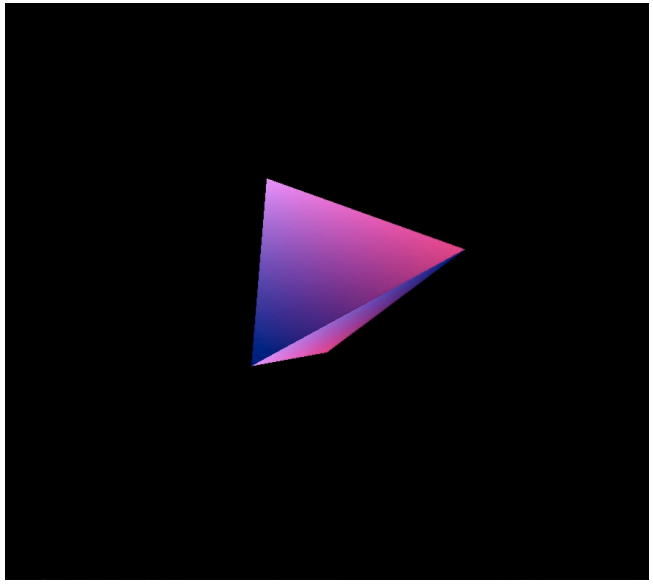
Koch Curve in Higher Dimensions

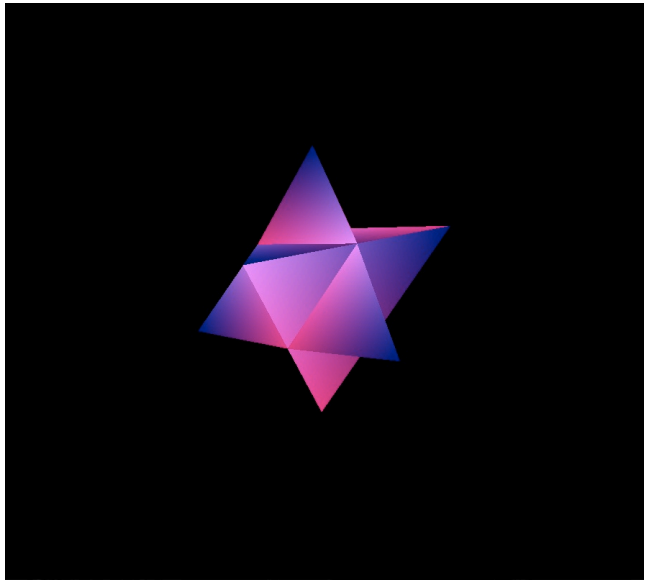


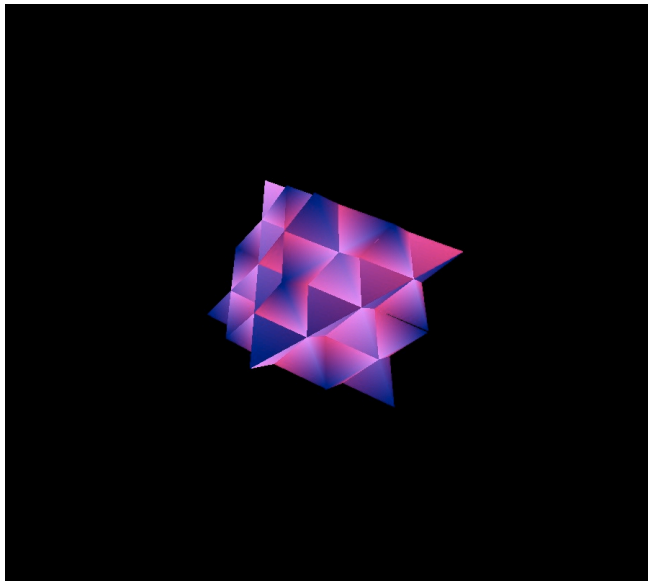
Tetrahedron

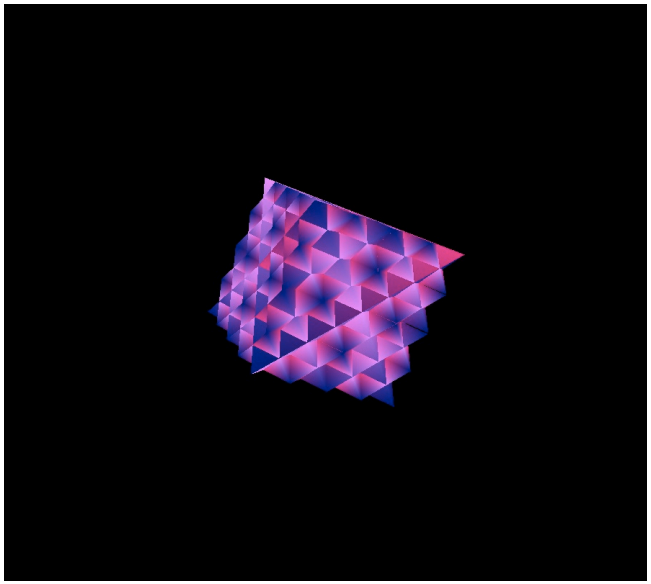


http://orderinchoas.files.wordpress.com/2013/05/sierpinskytrianglewikipedia.gif

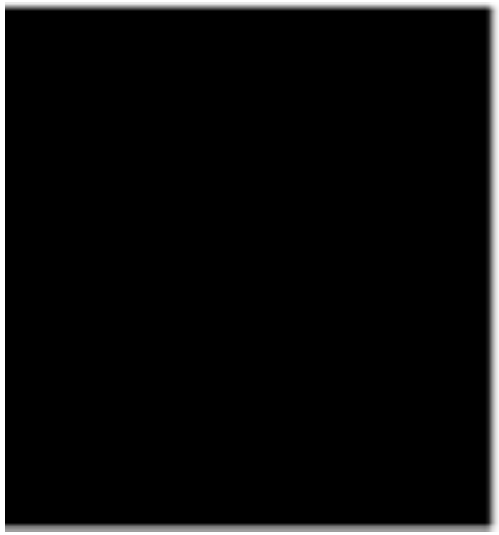


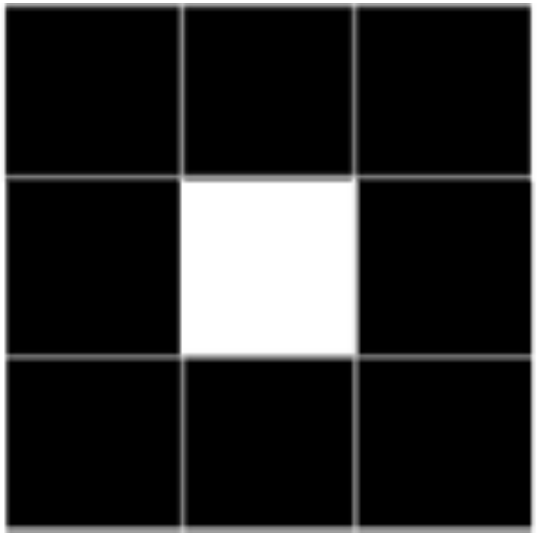


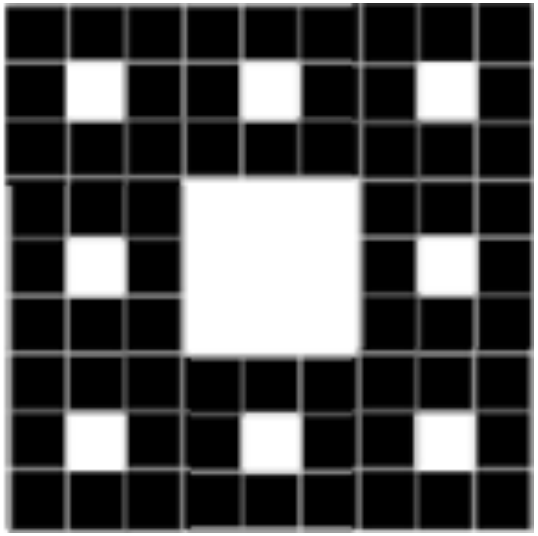




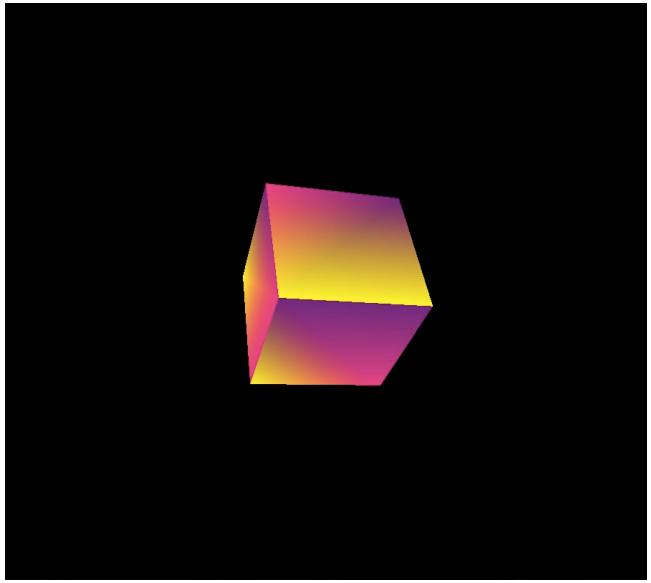
Cube

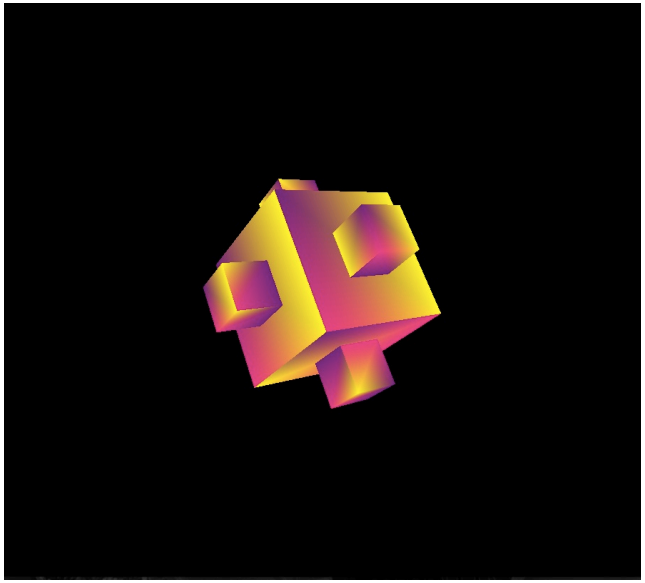


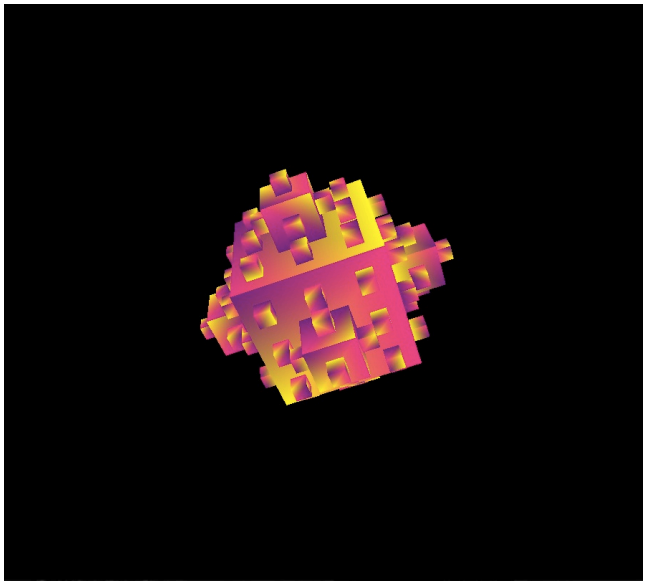


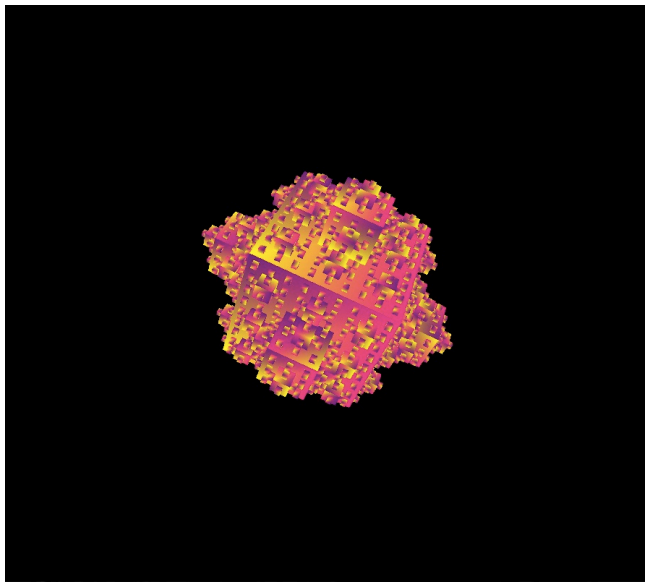


http://theiff.org/images/menger/menger1.gif



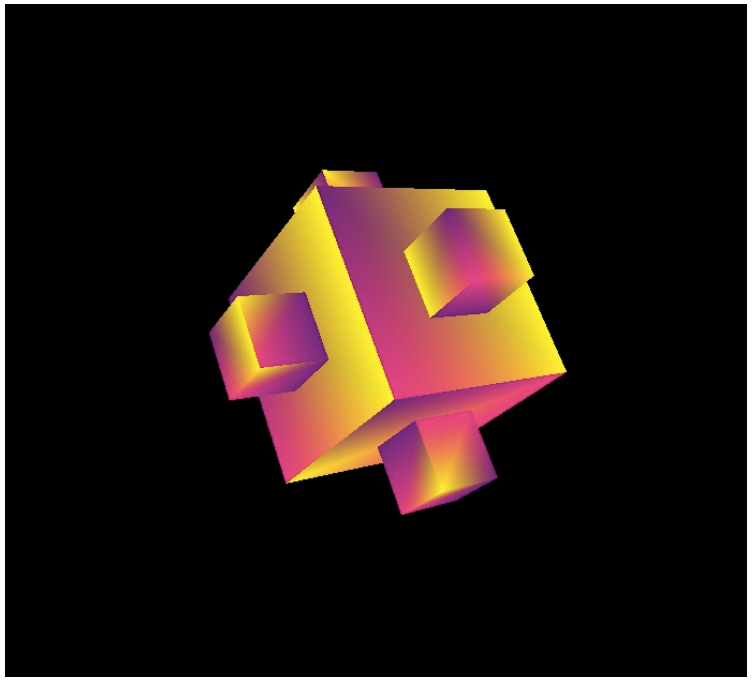




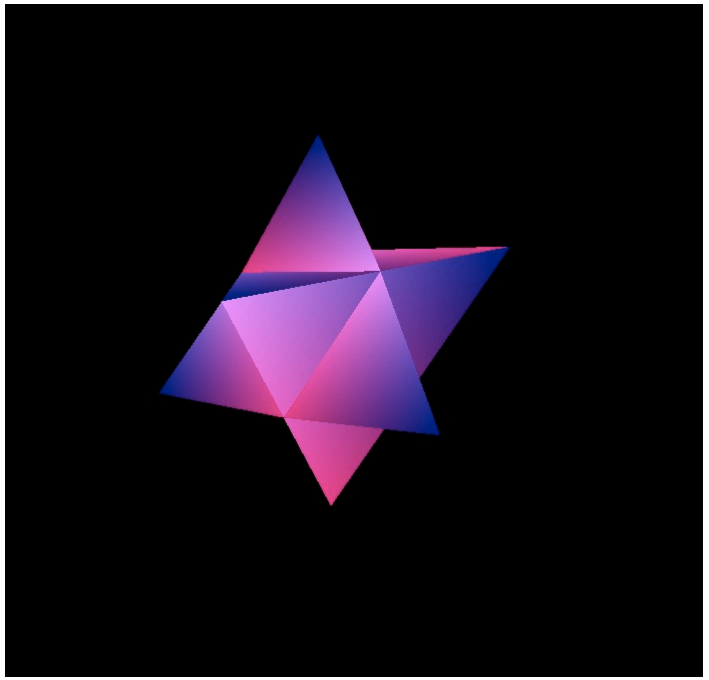


Fractal Dimension: Surfaces

$$D = \frac{ln(S)}{ln(\frac{1}{r})}$$

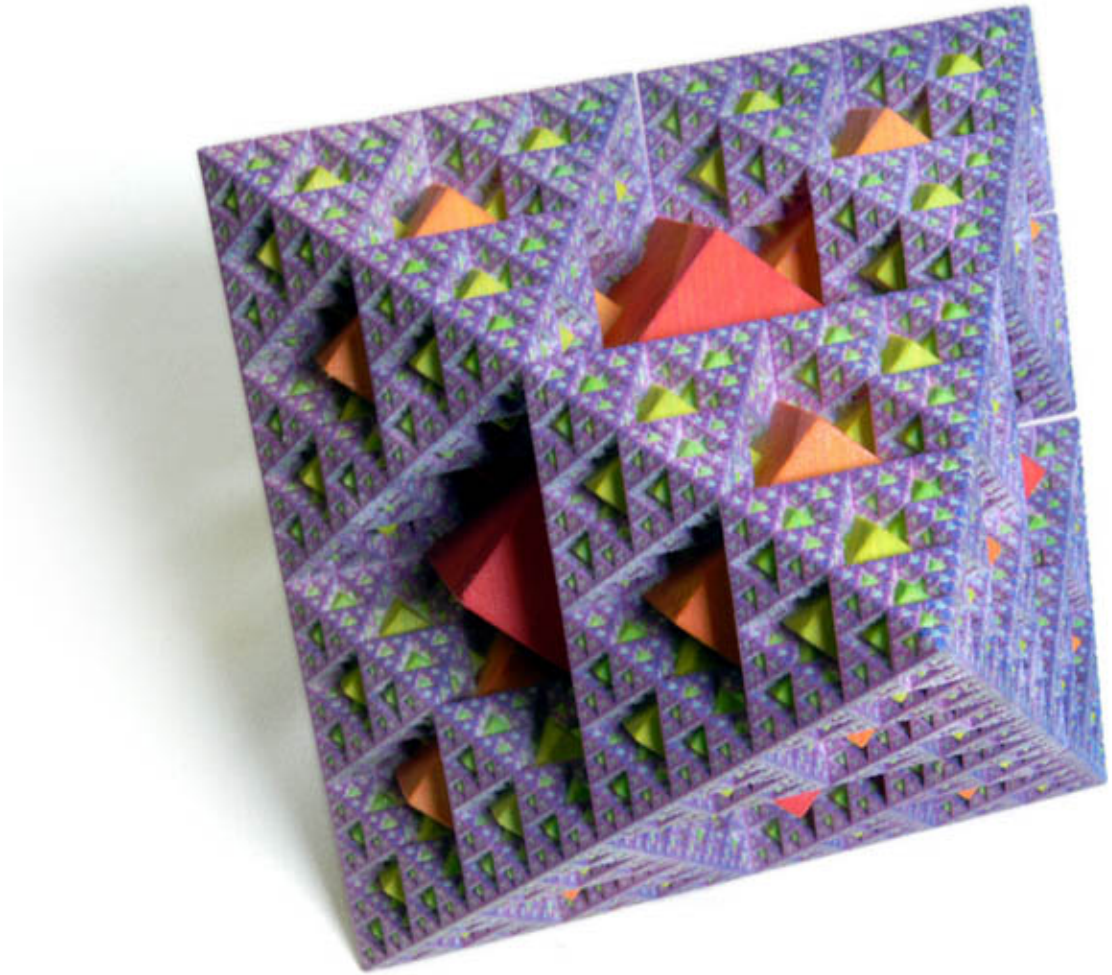


D=log13/log3



D=log6/log2

Convergence



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