

MATH 199 Project Proposal

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1 Proposal

Ulam's Spiral is a visual representation of the prime numbers typically seen statically as an image file. The spiral is interesting in number theory in that when the spiral is drawn with a large number of primes lines begin to become visible in the image. More interesting perhaps is that these lines can be identified with a quadratic equation like such:

$$f(n) = 4n^2 + bn + c \quad (1)$$

For certain values of b and c is found to have a higher density of prime numbers when compared to any random set of integers. The number of primes less than was asserted by Hardy and Littlewood to be represented by

$$P(n) \sim \frac{1}{\sqrt{a}} \frac{\sqrt{n}}{\log n} \quad (2)$$

The value of A depends on a,b,c and not n. Higher values of A imply that the set of numbers generated by that equation will contain a greater number of primes than any given random set of integers. When A is 1, the equation gives the expected probability of an integer being prime in a random set of numbers less than n. Finding values of b and c which give a greater value of A results in some quadratic equations that generate a greater number of primes than normal and hence form lines in Ulam's Spiral.

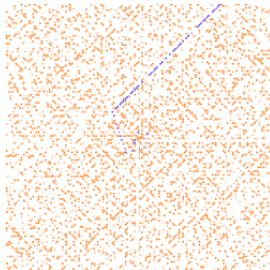


Figure 1: The line of primes generated from the equation $4x^2 - 2x + 41$

For my project I intend to make a 2-D representation of the spiral being generated in real time. This will include multiple modes, one where it only shows the primes appearing in real time and another where the number spiral is shown and the non primes are deleted. It will also highlight different prime numbers based on an input quadratic. This will be done first using Tkinter and have variable speed and generate in real time. Given reasonable time I will also attempt to implement it using HTML5 and Canvas in order to put it on my website and make it usable by anyone who visits it. I will generate primes using a modified version of the Sieve of Eratosthene. I will implement the following pseudocode:

Input: an integer $n > 1$

Let A be an array of Boolean values, indexed by integers 2 to n , initially all set to true.

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for i = 2, 3, 4, ..., not exceeding n:
  if A[i] is true:
    for j = i2, i2+i, i2+2i, ..., not exceeding n:
      A[j] := false
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Now all i such that $A[i]$ is true are prime.

Along with my own algorithm for generating a number spiral in order to selectively display pixels based on whether they are a prime or not. Once this is done, I will use the generated data to create a few options to aid with visualizing the spiral.

References

- [1] Guy, Richard K. *Unsolved Problems in Number Theory*. Springer, p. 7 3rd Edition, 2004.