Stability of 3D Quasicrystals: Advanced

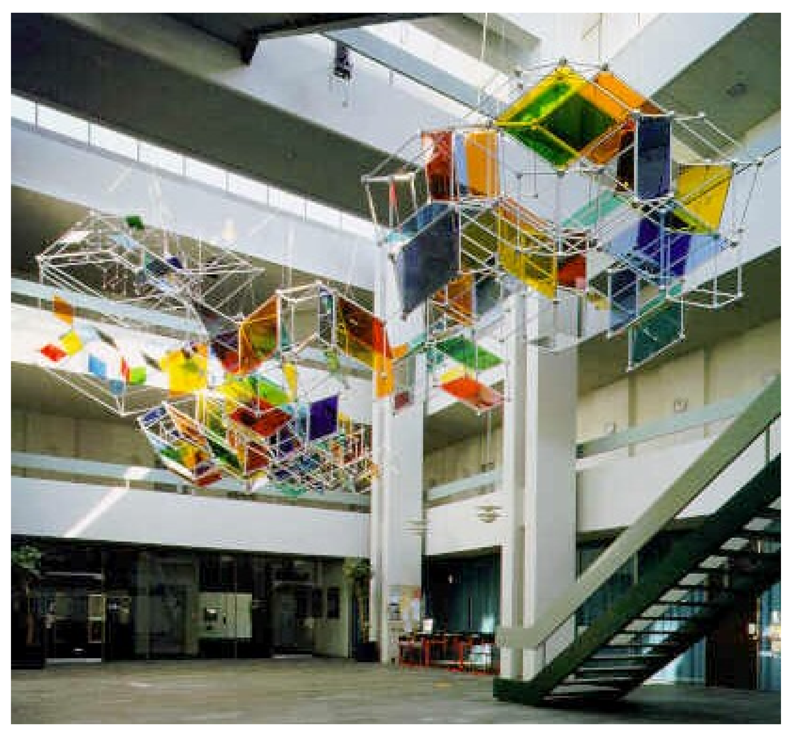
Arturo Guerrero, Yijing Chen, Zachary Berrabah, Eliana Duarte, Nima Rasekh, George Francis

Introduction

Motivation

The project is part of a greater effort to understand the stability of 3D quasicrystals. The goal is to figure out which parts of a given quasicrystal we have to make rigid such that the whole shape cannot be deformed. This was motivated by Tony Robbin, who needed this theory for his artwork (COAST) and in his case had to find the answer using experimentations.

COAST



In this 3D quasicrystal, Tony Robbin chose several rhombohedra and made them rigid by plating some of their faces. This had the effect of making the whole structure rigid. Rigidity means that the only possible motions of the structure are rotations or translations. It remains an open problem in mathematics to give criteria to decide when a 3D rod and pinion framework is rigid.

Figure 1: *Tony Robbin, COAST, Installed 1994 at Danish University, Destroyed!!!*

The 2-Dimensional Case: Wester's Theorem

Theorem 1: (Wester's Theorem, [2])
Let K be a rhombic carpet with associated Wester graph Γ and let Φ be a subgraph which is both spanning and connected. Then, bracing the rhombi corresponding to the edges of Φ makes K rigid.

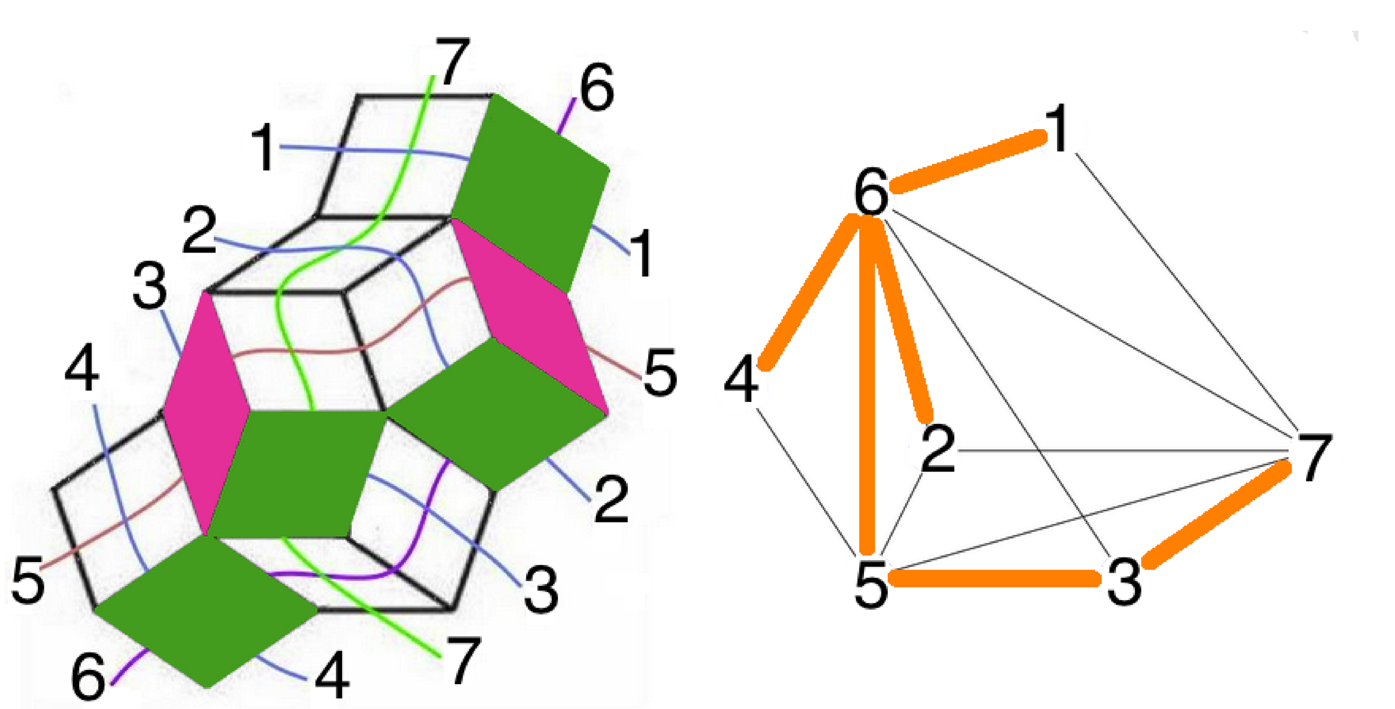


Figure 2: *A 2D quasicrystal rod and pinion framework is rigid if and only if the colored graph on the right is spanning and connected. Each edge on the graph corresponds to a plated face of the 2D framework on the left*

Goal

The goal is to write codes that can help us visualize 3D version of Wester's Theorem.

Improving Visualization of Rigid Bricks

One important stepping stone on proving 3 dimensional version of Wester's theorem is recognizing which parts have to be made rigid, which can be hard to visualize in 3D, which has been addressed by team member *Yijing Chen*.

Improvements

1. Add 3 projection planes to the original *Wobble.html* code to better visualize 3D.
2. Making a cube rigid will make the corresponding squares ridig as well.
3. Adding a rotator so that each side can be observed based on need.

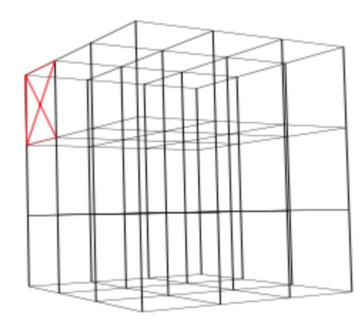


Figure 3: *Simple Cube from Wobble*

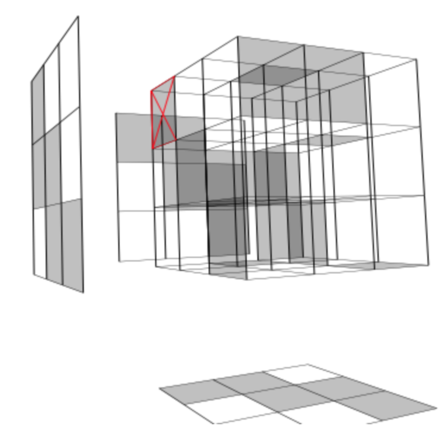


Figure 4: *Wobble with projections*

Visualizing Polyhedra

An important part of working with quasicrystal bricks is to visualize various rhombic polyhedra. Team member *Arturo Guerrero* wrote a program that helps us visualize such shapes.

Improvements

1. Rewrote large parts of original wobble code.
2. Manually found right coordinates for various polyhedra.

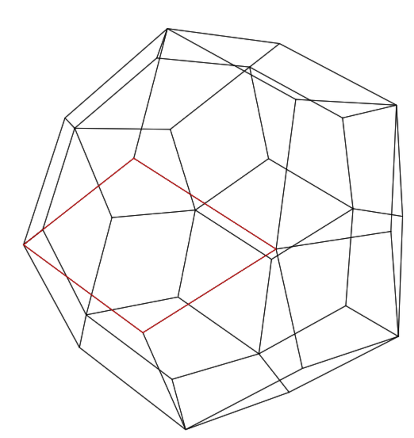


Figure 5: *Triacontahedron*

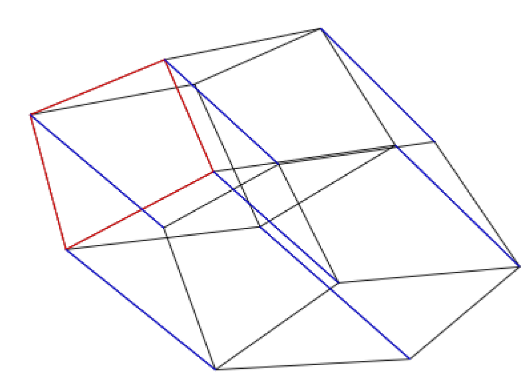


Figure 6: *Dodecahedron*

Rewriting Graphic Visualization Software

One of the big issues with the 3 dimensional construction of bricks is that they are really projections of six dimensional objects. A former student, *Geoff Ehrman*, has worked on a code, nqc.py, which allows us to visualize these 3D projections. However, the code employs the elaborate graphics toolkit Syzygy (szg), which does not work on any computer. Thus, team member *Sasha Lamtyugina* set out to solve this issue.

Improvements

1. Understand the main functionalities of the code and rewrite it into OpenGL, which can be run on almost any computer.
2. Significantly cutting down on required space and writing a user friendly code that can now easily be improved.

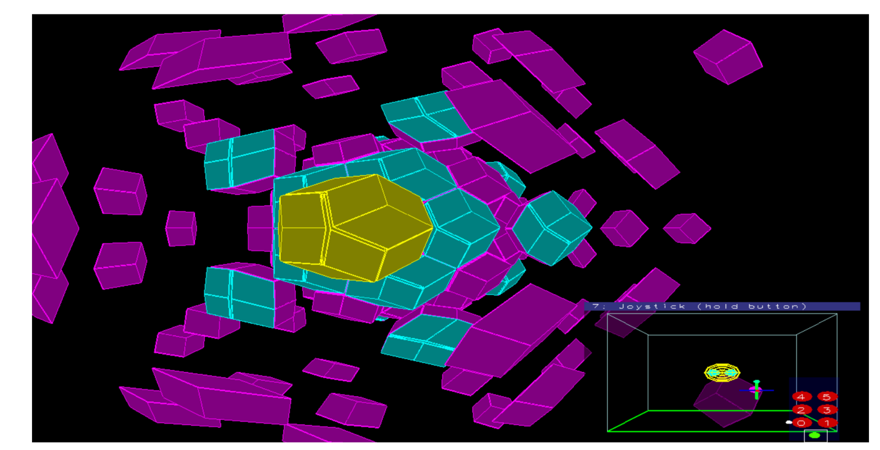


Figure 7: *Original code using szg library*

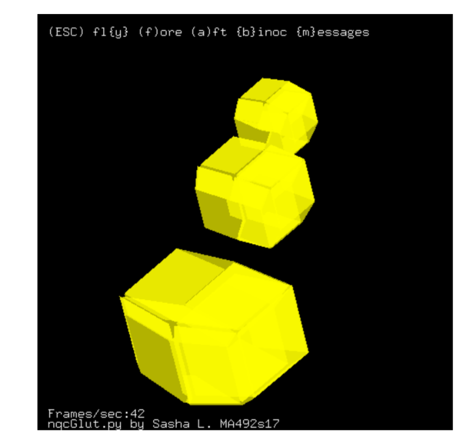


Figure 8: *New code using only OpenGL (no szg) showing 6D projections in 3D*

Visualizing Rotations of the Cube

One of the important tasks to analyze the 3D version of Wester's theorem is write a program that can witness the most complicated deformations of the cube, which is exactly what team member *Zachary Berrabah* worked on, by writing a program that deforms a cube in several different ways.

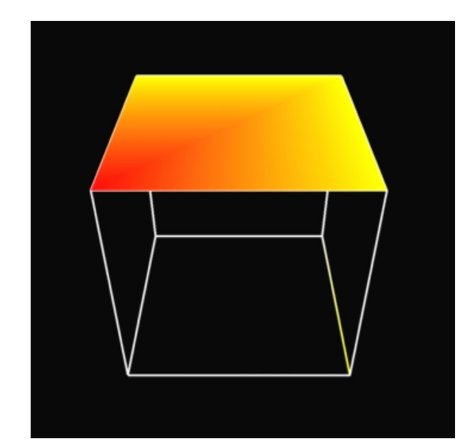


Figure 9: *Simple Cube*

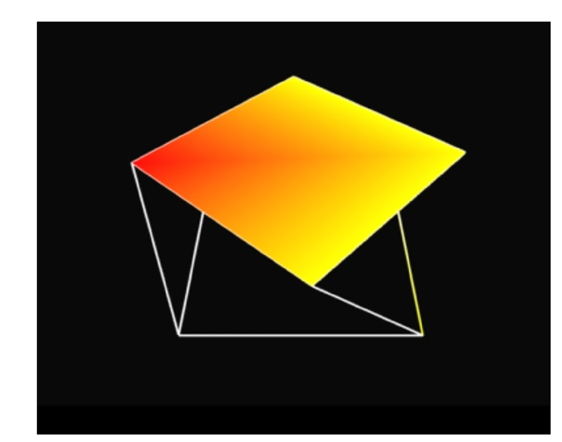


Figure 10: *Rotating Cube*

Future Directions

1. Produce a real time interactive computer animation (RTICA) to visualize quasicrystals.
2. Create an RTICA to model deformations of rhombic clusters.
3. Extend the functionality of the 3D cubical framework RTICA wobble.html.

References

[1] David Eck. *Introduction to Computer Graphics.* Online: http://math.hws.edu/graphicsbook/ (2016)

[2] Gélvez, Eliana M. Duarte, and George K. Francis. *Stability of Quasicrystal Frameworks in 2D and 3D.*

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