Post-Modern Geometry: In Lieu of an Introduction*

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1 Introduction

This lesson is an update of the corresponding lesson in the 1995 printed notes.

2 About this Course

This course is a favorite of geometers because it is one of the few to present an entire subject in one semester. It is generally taught in the style of a course on the history of ideas. This is also true of Math 306 which treats the history of the calculus. As such, it is a course in which strong opinions on how it should be taught are held, and so there cannot be a universally acceptible textbook, to be used year after year.

Math course numbers were inflated by 100 in the late nineties. Thus 306 is now 406.

Since 1980, but especially in the early nineties, I was strongly tempted to write my own textbook, at least for my editions of the course. But this

*From *Post-Euclidean Geometry: Class Notes and Workbook*, UpClose Printing & Copies, Champaign, IL 1995, 2004

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is a time-consuming, and not entirely good thing to do. Students should be guarded from learning the same slant from both major sources: text and lecture. However, over the years, a good set of notes from my lectures has developed, which was extensively edited and extended by Aarti Swaminarayan, who was my TA in 1993.

Serendipity brought a succession of good textbooks to my attention. The text for Math 303 in 1994, by David Kay[1] had the pleasant feature that it is written in the style now customary for high school geometry texts, but at a level that is challenging to a college student. For students who later will teach high school geometry[2] this was a welcome bonus. The book is an excellent reference book, treating many more topics than we could cover. Its emphasis on traditional Euclidean geometry nicely complemented my emphasis on non-Euclidean geometry. However, it was the consensus of the class that it was not essential for the course. So no text is required to supplement these notes.

Early in the first decade of the third millennium Michael Hvidsten's remarkably complete textbook appeared, together with a superb software package, *Geometry Explorer* (GEX) for Euclidean and hyperbolic (non-Euclidean) constructions. This text so nearly paralleled my own concept of the course, that I have redesigned my course around this text. You may consider my notes elaborations of Hvidsten's text, or you may consider his text the best supplementary reading available.

Most importantly, in 2008 Mike revised and extended his Geometry Explorer construction kit with an elliptic plane (spherical geometry), two bounded models of the Euclidean plane, the Klein-Beltrami and the upper half plane (UHP) models of hyperbolic (non-Euclidean) geometry along with the original Euclidean and Poincare (conformal) models. The user manual for GEX 2.0 is practically a free standing text book of its own.

If you are studying these notes independently of my course you are encouraged to acquire some textbook on geometry, even second-hand, also as a reference for things that slipped your memory since high school.[3]

[1] D.C. Kay, College Geometry, Harper Collins, New York, 1994.

[2] And as parent, aunt, uncle, guardian, friend of a high-schooler someday, all of you will have an opportunity to help a youngster learn geometry.

[3] H.S.M. Coxeter, F.R.S., Introduction to Geometry, John Wiley & Sons, 1961, 2nd ed. 1969.

3 What is the Name of this Course?

Math 302/402 is often called *Non-Euclidean Geometry* because this subject became enormously interesting 200 years ago when people discovered that Euclid's geometry, the only one known and accepted as true for 2,000 years, wasn't the only one. This discovery took a little over 100 years to be completed. Around 1900, Hilbert's axiomatic system closed the books on this matter. By the time of the Great Depression, Birkhoff worked out a system for implementing this non-Euclidean geometry in the secondary schools. In the fifties, I still learned geometry from a textbook no more cognisant of geometrical evolution than contemporary creationists are of the neo-Darwinian synthesis. My children used a different, thoroughly modern textbook in their highschool. So, I would prefer to consider our course to be one on *post-Euclidean geometry*, preparing the way of teaching geometry in the new millennium.

The course is now taught in the collaborative spirit of the mathematics reform movement. That is, students are not only allowed to work together, but expected to collaborate in the solution of non-trivial problems. Thus, the problems posed as "questions" in these notes generally require more thought than the more routine exercises of the typical calculus course. In addition, this course stresses the uses of accurate drawings, executed by hand and computer. For this purpose there are three documents in the Appendix. The first was prepared by the study-group "Descartes" in the 1993 edition of this course. The second was prepared by Aarti Swaminarayan at the same time she herself was learning to use `xfig` to draw the figures in these notes. The third is her project on `mathematica`.

This and the next paragraph pertain to the 1995 notes.

To emphasize the role that pictures play in the learning and teaching of geometry these Class Notes are also a Workbook. It comes with no illustrations, and generous margins for you to supply them. Ample examples, hints and instructions will be given in class on how you should do these illustrations.

The web documents replacing the handouts do have some figures now, but you should add many more, perhaps on the backside of any print-outs.

M.J. Greenberg, Euclidean and Non-Euclidean Geometries, W.H. Freeman, 1974.

G.E. Martin, The Foundations of Geometry and the Non-Euclidean Plane, Springer Verlag, 1975.

E.C. Wallace and S.F. West, Roads to Geometry, Prentice Hall, 1992.

D.W. Henderson and Daina Taimina, Experiencing Geometry, Prentice Hall, 2001, 2004.

Michael Hvidsten, Exploring Geometry, McGraw Hill, 2005.

There will be handouts for many of them, and you can trace or paste these in. Drawing them yourself will be of much greater benefit to you.

Finally, the course is designed to be learned best as an active member of a small group of students rather than by working alone. It is essential that you join such a group and plan on studying regularly with your work-group. The small sacrifice in adjusting your schedule and your effort to develop the self-discipline required to work with others will be of inestimable benefit to your education. Write the names telephone numbers and e-mail addresses of your fellow group members here.