1. Seminar of Limits of Sequences,

Revised 5apr16.

This is a continuation our collection of problems that illustrate the ideas we have been developing over the past 2 weeks. We will discuss their solution “in-seminar”, which means that I expect class participation. See syllabus.

Problem 1. Suppose that the sequence θ_n is **not** a null sequence. Show that this statement is false, and correct it *There exists* $\varepsilon > 0$ *so that for all* n, $|\theta_n| > \varepsilon$*”*

Problem 2. Show how every irrational number is the limit of a (weakly) decreasing sequence of rational numbers. (Weakly decreasing means $x_{n+1} \leq x_n$.)

Problem 3. Suppose $a_n \to a$ and $b_n \to b$, then determine whether the following are true or not. If true, prove it. Else find a counterexample, correct and prove the statement.

3a If $a < b$ then $(\exists N \in \mathbb{N})(n \geq N \Rightarrow a_n < b_n)$

3b If $a \leq b$ then $(\exists N \in \mathbb{N})(n \geq N \Rightarrow a_n \leq b_n)$

3c The $\lim_{n\to\infty}(a_n - \frac{1}{b_n}) = a - \frac{1}{b}$.

Problem 4. Find and verify the $\lim_{n\to\infty} x^n$ for $x \in \mathbb{R}$.

Problem 5. Determine the limiting behavior of:

$$\frac{x^n}{x^n - 1}, \frac{1}{x^n + 1}, \frac{1}{x^n - 1}, \frac{1}{x^n + x^{-n}}, \frac{1}{x^n - x^{-n}}.$$