348M1 - lesson

Note Title 8/26/2008

Liebeck Ch1 & Symbolic Logic
This week the goal is to review/learn the symbolic vocabulary of mathematics

See L pg199f for a table of symbols used in Liebeck
We add a few more to conform to MA347 and to sharpen the language of proofs.

Review set notation (L pg1)

$\{1, \{2\}\}$, $\{x \mid x \in \mathbb{R} \wedge x^2 < 2\} = \{x \in \mathbb{R} \mid x^2 < 2\}$

the set of all x — x is real — and

emptyset $\emptyset = \{\ \}$

Def: A proposition is a sentence that is either T or F

(Def: A sentence has a subject and a predicate.)

Examples — Prop?

The king of France is bald. (Russel) — No

Math is hard. (Barbie) — No

$x \in \mathbb{R} \wedge x^2 < 2 = P(x)$, $P(1)$ is true, $P(10)$ is F

Def: A set S is a collection of objects x for which $x \in S$ is a proposition

$S = \{x \mid x \in S\}$

Set Theory

Sets $A, B, \ldots$ elements $x, y, \ldots$

sets $A \cap B = \{x \mid x \in A \ \wedge \ x \in B\}$ intersection

$A \cup B = \{x \mid x \in A \ \vee \ x \in B\}$ union

$A \setminus B = \{x \mid x \in A \ \wedge \ x \notin B\}$ complement

props $A \subset B \sim (\forall x)(x \in A \Rightarrow x \in B)$ subset

Propositional Logic

new symbols, for propositions $A, B, C \ldots$

conjunction $A \wedge B$ ... and ...

disjunction $A \vee B$ ... or ...

caution, this is not the exclusive-or

as in A or B but not both A and B

$$A \times B \sim (A \vee B) \wedge \neg (A \wedge B)$$

negation $\neg A \sim \overline{A}$

Rules of logic: $\neg\neg A$ A double negative

$$\overline{A \wedge B} \sim \overline{A} \vee \overline{B}, \quad \overline{A \vee B} \sim \overline{A} \wedge \overline{B}$$

material implication $A \Rightarrow B$ if A then B

Def $A \Rightarrow B \sim \overline{A} \vee B$

T1: See also webnotes "truth.pdf"

goal: using truth tables to check rules

$A \wedge B$

conj

T

F

A	B	$A \wedge B$
1	1	1
0	1	0
1	0	
0	0	

$A \wedge B$	0	1
0	0	0
1	0	1

looks like a multiplication table

dis

$A \vee B$	0	1
0	0	1
1	1	1

$A \times B$	0	1
0	0	1
1	1	0

$\overline{A \wedge B}$	0	1
0	1	1
1	1	0

$\bar{A} \vee \bar{B}$	0	1
0	1	1
1	1	0

! yes

proof of $\overline{A \wedge B} \sim \bar{A} \vee \bar{B}$

what is $\overline{A \times B} \sim$?

what is the truth table of

$A \Rightarrow B \sim \bar{A} \vee B$

$A \Rightarrow B$	0	1
0	1	1
1	0	1

What is $\neg(A \Rightarrow B)$?

Caution: it's not $A \Rightarrow \bar{B}$, for example

$\neg(A \Rightarrow B)$

$\sim \neg(\bar{A} \vee B) \sim \overline{\bar{A} \vee B}$

$\sim \bar{\bar{A}} \wedge \bar{B} \sim A \wedge \bar{B}$

$A \Rightarrow B$ material implication

$B \Rightarrow A$ converse

$\bar{A} \Rightarrow \bar{B}$ inverse

$\bar{B} \Rightarrow \bar{A}$ contrapositive

$\bar{A} \Rightarrow B$ See $\langle h.1.4 \rangle$ exercise in

$A \Rightarrow \bar{B}$ Liebeck

simplification to $\neg, \vee, \wedge$

imp $A \Rightarrow B \sim \bar{A} \vee B$

conv $B \Rightarrow A \sim \bar{B} \vee A \sim A \vee \bar{B}$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad ||$

inverse $\bar{A} \Rightarrow \bar{B} \sim \bar{\bar{A}} \vee \bar{B} \sim A \vee \bar{B}$

contr. $\bar{B} \Rightarrow \bar{A} \sim \bar{\bar{B}} \vee \bar{A} \sim B \vee \bar{A} \sim \bar{A} \vee B$

contrapositive $\sim$ proposition

proof "by contradiction" is usually a proof of the contrapositive.