

## The Structural Morphology of Penrose and Quasicrystal Patterns Part I



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### KEYWORDS

Penrose, quasicrystals, lattice structure, duality, random patterns

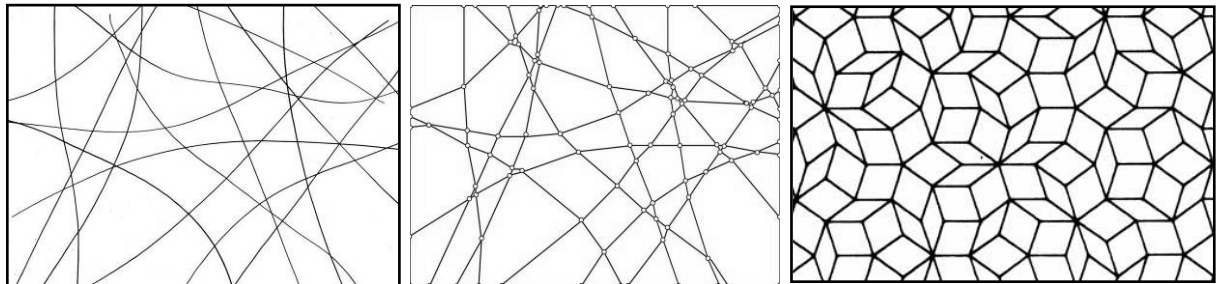
### PAPER

The paper will briefly give an introduction to the structural properties of randomly generated patterns of intersecting 1D and 2D elements in 2D and 3D space. The conclusion is that these random configurations create patterns for basic structural types as lattice and plate structures. This issue has already been discussed and proven in [Wester 2003 and 2004]

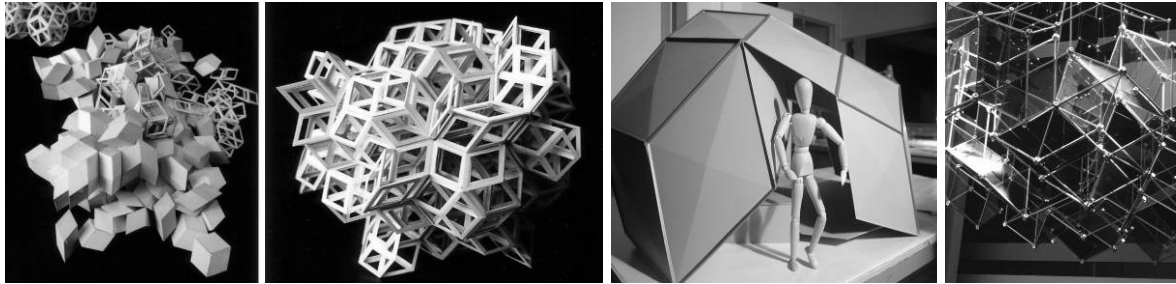
In this paper I will concentrate my investigations on the structural behaviour of the relatively recently discovered - partly chaotic and partly orderly - geometries as Penroses (found by Roger Penrose in 1970<sup>th</sup>), described in [Gardner 1989] and quasicrystals (found by Daniel Schechtman in 1980<sup>th</sup>), described in [Senechal 1996 and Robbin 1997]. These new geometries give almost unlimited possibilities for configurations and shapes in 2D and 3D, geometries which can form single and double layer faceted domes, two or multilayer space trusses as well as 3D mega structures – all as pure lattice or pure plate structures or a combination of these. The topology of these geometries turns out to be duals to the random patterns mentioned before and described in [Wester 2004], following the topological duality and embedded in the **Euler-Descartes theorem** for the topology of polyhedra and polytopes:

$$V - E + F - C = K$$

linking the number of Vertices (0D elements), Edges (1D elements), Faces (2D elements) and Cells (3D elements) - and K is a constant, depending on the genus of the configuration [Coxeter 1973].



**Figure 1: A random 2D pattern of 1D elements (represented by human hair) (left) and the similar planar bar-and-node counterpart (centre) and a dual penrose configuration (right). Centre and right have similar rigidity conditions as lattice structures.**



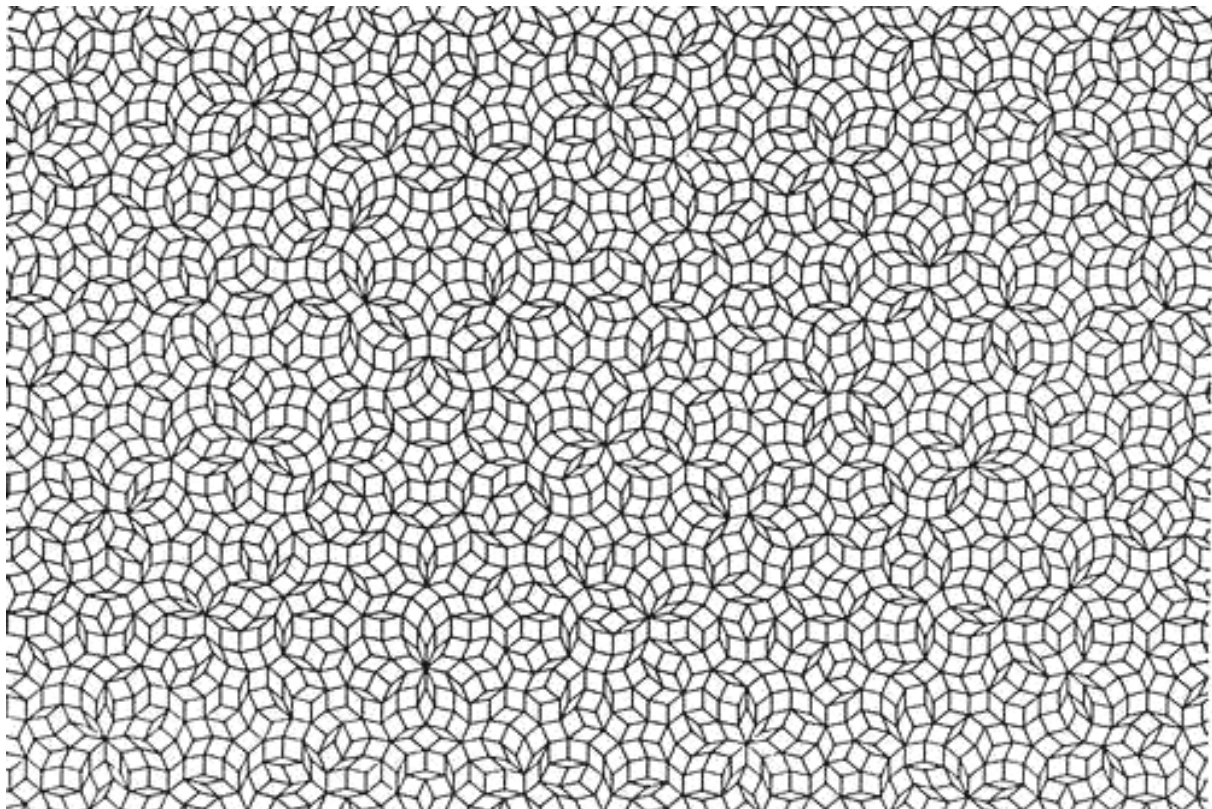
**Figure 2: The puzzle of combining the two building blocks to a close packing quasicrystal system (two left). Examples of 2D and 3D architectural quasicrystal structures (two right).**

The concept of the structural duality [Wester 1984 and 1997] discovered by the author in 1976 follows the topological duality, and this gives a key to the understanding of the structural behavior of the penrose and quasicrystal geometry. It is the first time that the rigidity of these semi-chaotic structural configurations have been described.

The quasicrystal geometry seems to imply many potential possibilities for architectural structures [Weinzierl and Wester 2001], not least because they easily can adapt to function, landscape, different spans etc, using a minimum of different structural elements: In a configuration you will find only one length of bars (1D) making up only one type of facet (2D), enclosing only two types of cells (3D) all joined by only one type of nodes (0D).

The system, so far neglected in architecture, seems to contain a huge amount of unrealised possibilities for architectural expressions because of the simultaneous morphological simplicity and complexity.

## 2 The Rhombic 2D Penrose Pattern as a Plane Lattice Structure



**Figure 3: How many bracing bars are needed for rigidity? – and where to put them?**

The rules of duality tell us that vertices swap with facets while edges remain the same number (see the Euler theorem above where  $C$  is 0 or 1 in 3D). The result is that a penrose composed of rhombs is dual to the pattern formed by random intersection of 1D elements (fig. 1 left) which consists of 4-valent vertices. This implies that the average vertex in a large penrose is 4-valent i.e. the penrose fulfil internally the necessary number of bars to nodes for rigidity and the number of bracings is related to the border [Wester 2004].

As the rhomb-based penrose pattern consists of two different rhombic 4-gons, it will be possible to identify bands (so-called “ribbons” or “pasta-bands”) through the pattern. These ribbons cut the parallel edges of the rhombs. Because of the underlying pentagonal symmetry we find parallel ribbons in the five directions. If all ribbons are marked then they form a pattern crossing each other in 4-way vertices and they will include all rhombs even the ribbons are not identical. The parallel lines crossing the ribbons belong to the orderly part of the penrose pattern while the differences of the individual ribbons belong to the chaotic part, see fig.4.

Observations show that if one ribbon is fully braced hence rigid, then it is possible to extend the rigid domain as non-braced rhombs by the rule: “one new node is fixed by two new bars”. In this way the entire domain from the braced ribbon until the next parallel non-braced ribbon is rigidly connected to the braced ribbon. This is valid for both sides of the braced ribbon. To continue it is necessary to brace only one rhomb in the neighboring parallel ribbon. Then the whole ribbon has become rigid and hence is the seed for the rigidity until the next parallel band, etc. See fig.6, left.

As indicated above then the average vertex for a penrose is 4-valent. The “regular version” of this could be the 2D pattern of squares which have the same characteristics. The (two straight) ribbons in the square net cross each other and cut parallel edges. This property is used for the method by means of a subgraph to determine the number and position of bracing bars to evaluate if such a pattern is rigid or not [Baglivo and Graver 1983]. The proofs for the stability condition for the square net and the rhombic penrose are similar because the possible movements (gliding) of the parallel bars crossing the ribbons are the same. Where the square net operates with numbered rows and columns, the penrose operates with the five rows of different orientated ribbons called a,b,c,d,e all numbered according to the selected part of the penrose. The minimal number of bracing bars is the total number of ribbons minus one – just as for the square net. The five different systems of ribbons have the same ability to glide as the two directions in the square net.

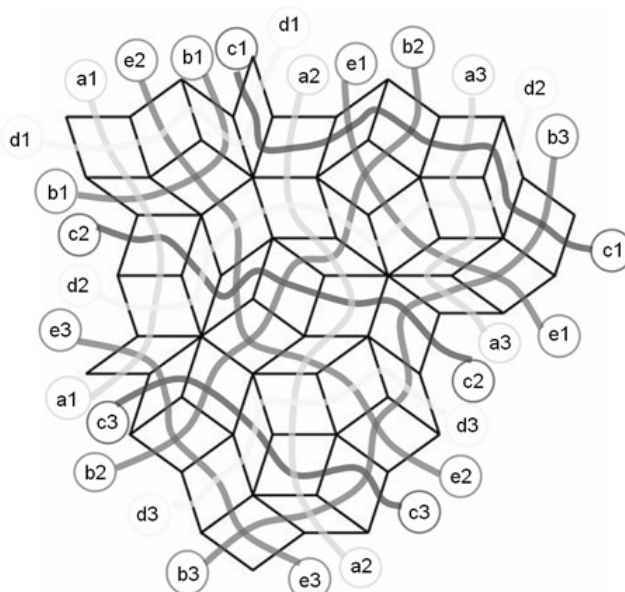


Figure 4: Penrose with 3 x 5 ribbons

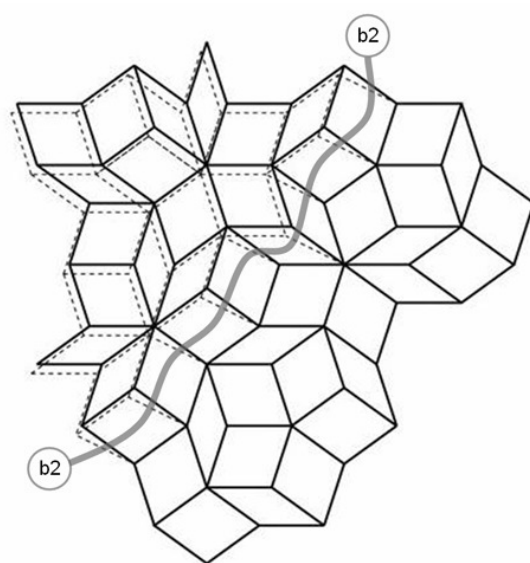
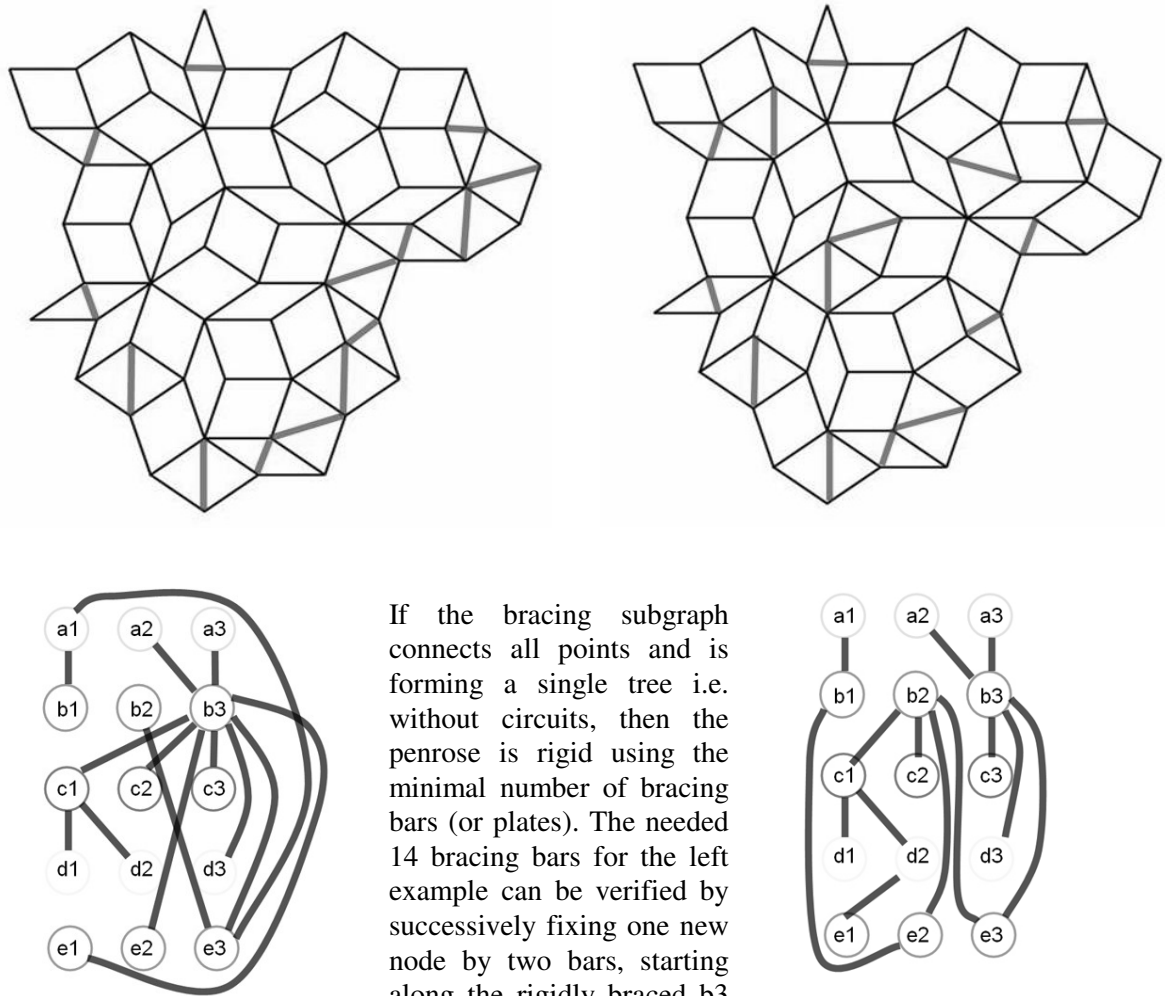


Figure 5: The b2 gliding ribbon.



If the bracing subgraph connects all points and is forming a single tree i.e. without circuits, then the penrose is rigid using the minimal number of bracing bars (or plates). The needed 14 bracing bars for the left example can be verified by successively fixing one new node by two bars, starting along the rigidly braced b3 ribbon.

**Figure 6: Braced penroses and their subgraphs**

### 3 The Quasicrystal Pattern as a 2D Lattice (Surface) Structure in 3D

Imagine a large quasicrystal cleaved into two halves and use the surface as a single layer structure. The rigidity of such shapes follow the rules for polyhedral combined lattice and plate structures as described in [Wester 1991], i.e. for an unsupported closed polyhedron, all (rhombic) facets must be either braced into triangles with bars or with plates in all rhombs to be rigid, see fig.2c.

### 4 The Quasicrystal Pattern as a 3D Lattice Structure in 3D

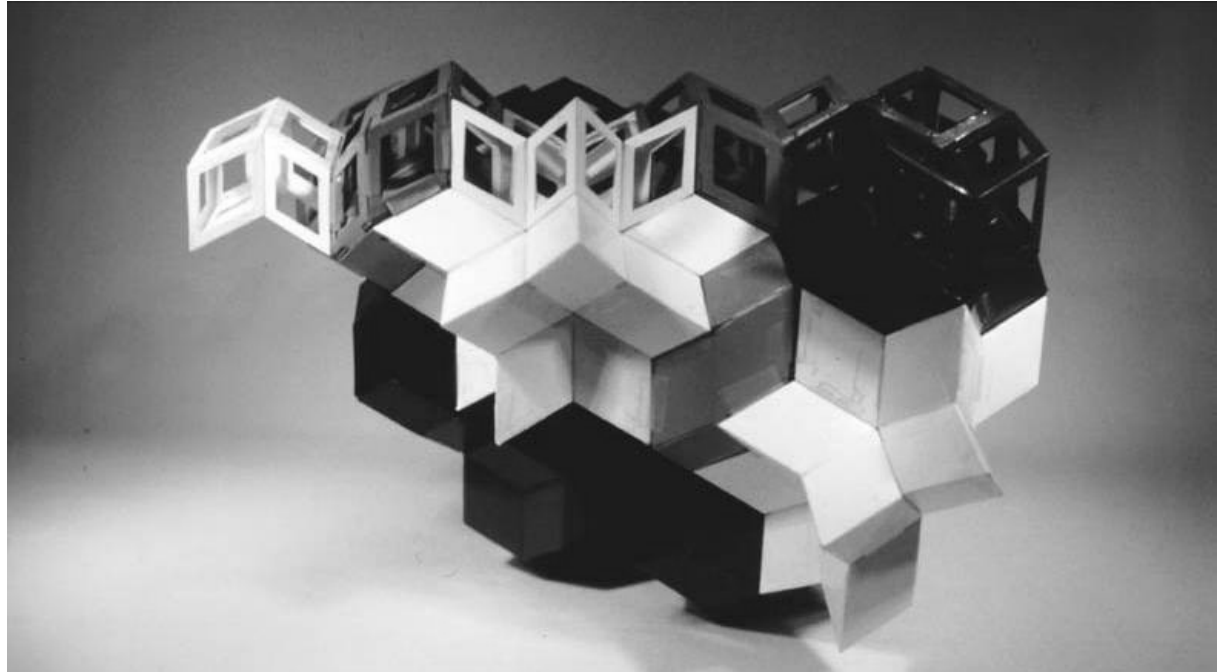
The rules of duality tells that the number of vertices and cells swap while edges and facets swap (see the Euler theorem above). The result is that a quasicrystal composed of golden cubes is dual to the pattern formed by random intersection of 2D diaphragms which consists of 6-valent vertices, only. This implies that the average vertex in a quasicrystal is 6-valent i.e. a “large” quasicrystal fulfil the necessary number of bars to nodes for rigidity [Wester 2004].

As penroses have ribbons with parallel bars, the quasicrystals have double layers separated with parallel bars (sandwiches) of similar regularities. Six sets of such sandwiches intersect each other with the same kind of parallel qualities as the five sets of ribbons in the penrose. One could expect that the similarity between the penrose and the quasicrystal as lattice structures implies similar operational methods for designing rigid quasicrystals, but this seems not to be the case. Such methods are still under discussion. Until useful methods are found then the rule that: On the basis of rigid 3D

elements, the step-by-step method: “three new bars is needed to fix one new node” can be used to successively establish a rigid 3D quasicrystal structure.

The sandwich seems to be an interesting alternative to space trusses as it has possibilities in its functionality by adaptation to small and large rooms, to the landscape, high and low areas etc.

The content of chapter 3 and 4 will appear and be discussed further in part II of this paper, which will be presented at a future conference.



**Figure 7: An architectural quasicrystal composition.**

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