The Visual and Structural Properties of Quasicrystals

Key words: Quasicrystal Sculpture, Quasicrystal Architecture, Stability

A Quasicrystal for Cherry Valley

A visually rich and complex quasicrystal sculpture is quickly assembled with relatively few standard parts of only three types.

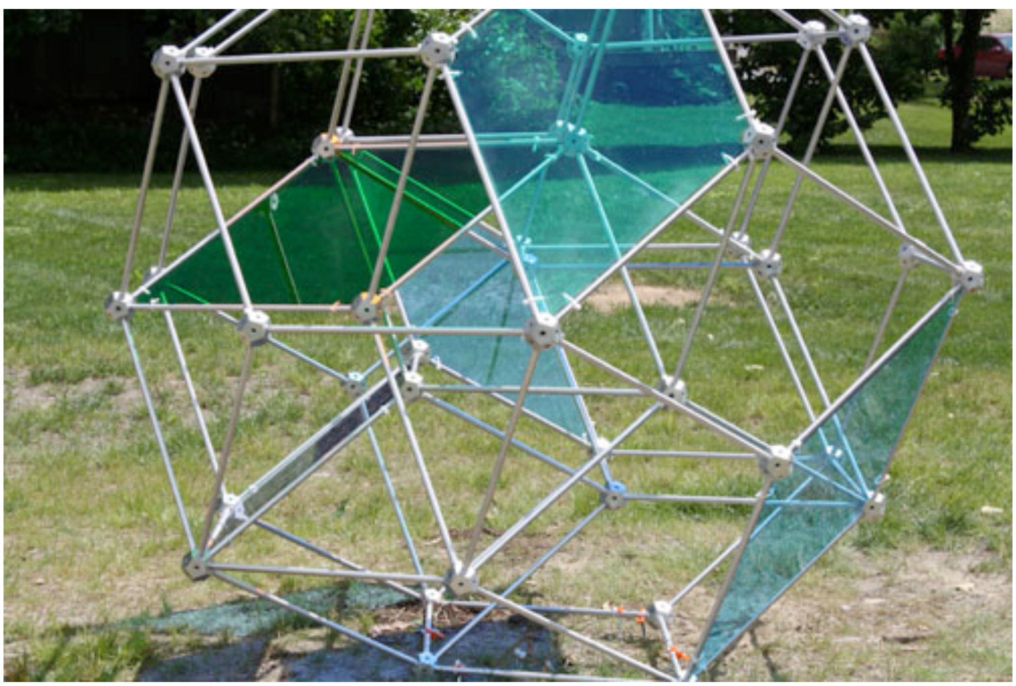
Quasicrystals fill space with a non-repeating pattern; parts repeat, but not at regular intervals. In two dimensions, the pattern might be a Penrose tessellation, although other similar patterns could also be in this category. In three dimensions, the units are two skewed cubes, and in a lattice structure these can be made with rods and dodecahedral nodes. All the rods are of the same length; all the nodes are the same and in the same orientation; all the faces of the lattice are the same rhomb, and can be filled with identical plates.

For the Cherry Valley Sculpture Exhibition of 2012, I made a quasicrystal sphere. It has a triacontahedral hull – a 30 sided figure that derives from the fusion of a regular dodecahedron and a regular icosahedron. Nested inside my hull is a rhombic icosahedron and nested inside that is a rhombic dodecahedron.

Even though all the parts are standard, the sculpture has 2-fold symmetry (of squares),

3-fold symmetry (of triangles and hexagons),

and 5-fold symmetry (of star pentagons), depending on the location of the viewer.

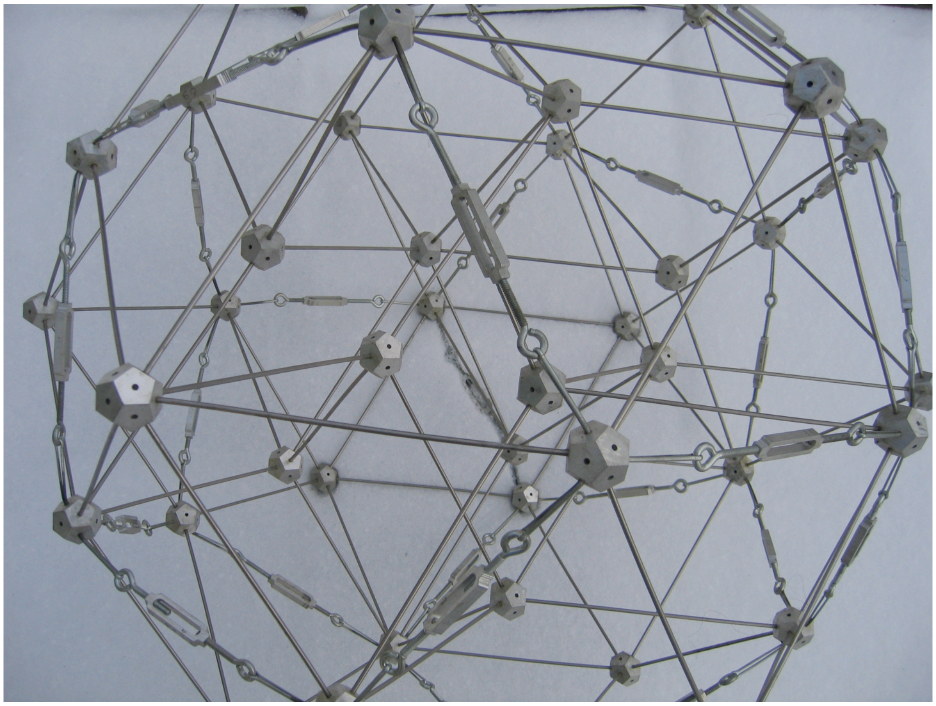


This wonderful complexity of aspect is also apparent in the shadows that the sculpture casts.

Structural considerations

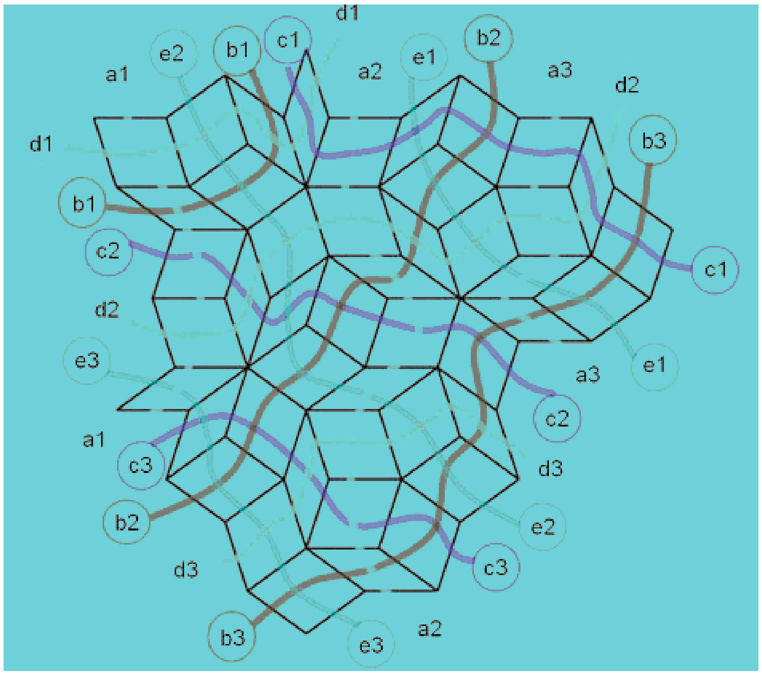
As an artist, I am primarily concerned with the visual properties of quasicrystals; for a wider application to architecture, however, the structural and rigidity properties of these structures must be understood. Two-dimensional and three-dimensional quasicrystals are composed of rhombs, which are not in themselves rigid. Solid dodecahederal nodes, such as I use, do provide some rigidity, but something must be done to establish an essential stability. In 1990 I began to study three techniques to made quasicrystals rigid: stress-skins, the triangulation of some rhombs, and quasicrystals as pate structures.

For the first option, I initially covered a quasicrystal ball with canvas pieces that were then seized with a plastic medium that shrank the material. Mathematical quasicrystals were first proposed as a model of a fluid because load applied to one part of the structure is not translated through the crystal but rather dispersed to the skin. While the surface canvas was tight, the canvas-covered quasicrystal ball was surprisingly strong – I sat on it, but as expected, the structure became flexible when the stress-skin loosened. The structural equivalent of a stressed-skin is shown below: every exterior rhomb is crossed by a turnbuckle placing the “skin” in tension. Again, the ball is rigid. Unfortunately, not many architectural designs can incorporate a continuous, complex, and positively curved skin, with a quasicrystal interior.

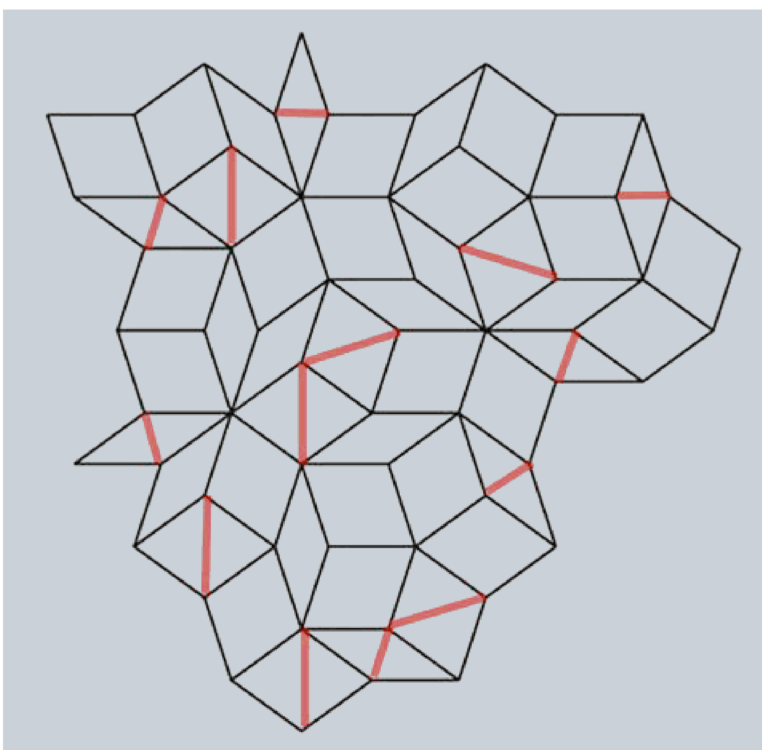


My ~~fiend~~ Ture Wester, an engineer well known to this audience, has studied the second structural strategy: triangulating the rhombs with rigid members. Since these triangles ruin the visual properties noted above, it is necessary to discover the fewest possible bracing members. In the two-dimensional case, Wester noticed ribbons of adjacent cells that all have parallel edges. In fact he noticed five sets of these ribbons, or one set of ribbons rotated at 72 degrees around a central point. These sets of ribbon-lines are a hidden structure of the quasicrystal that were first described by Robert Amman, and called "multigrids" by Nicolas deBruijn, 1980, who used them in his algorithm to generate quasicrystals. (In general, quasicrystals are projections of regular cubic cells from higher dimensional space – these Amman lines or multigrids are residues of the higher dimensional rectilinear grids.) Thus, structural considerations of physical quasicrystals are deeply related to their hidden mathematical structures.

Wester found that these ribbons of cells with parallel edges allowed him to treat the pattern as if it were a rectangular grid. As in such rectangular grids, once a ribbon is fully braced, the rigidity can be extended by the rule "one new node is fixed by two new bars." Following this well-known rule and its corollary, Wester could stabilize the pattern below with just 14 members: 15 total ribbons minus 1.



Wester’s analysis of the ribbon sub structure.



Wester’s minimal bracing of 14 bars.

Sadly, Ture died while still fairly young, and before he completed his examination of the three-dimensional case – the information really needed for architectural applications. For a large quasicrystal sculpture built in Denmark of 700 nodes, my engineering-candidate assistants and I intuitive placed acrylic plates to function like bracing bars, as also seen in the Cherry Valley quasicrystal. Like Wester's examination of the two-dimensional grid, we found that relatively few plates-as-bars were needed to make large aggregates stiff enough to be lifted up by a crane from a single point. (see YouTube video: http://www.youtube.com/watch?v=dFpinVoeNOc) However we still need a theoretical understanding of quasicrystal lattices, and since there are Amman structures in a three-dimensional quasicrystal, then called Amman planes, an analogous theory should be possible.

George Francis: your discoveries etc here

I also investigated making quasicrystals with plates, without nodes or rods. There is a wonderful economy of means with plate-structure quasicrystals: every plate is the same shape. It is a rhomb with an acute angle of tan τ: 1, or approximately 63.44 degrees. If the plates are to be sub-assembled into skewed-cube cells or half cells for subsequent assembly, then the plates could be beveled to ease that assembly. Only two sets (here called A and B) of beveled plates are necessary. The dihedral angles of bevel for plate A is as follows: 54 degrees for edges leading to the acute angles and 36 degrees for edges meeting at the oblate. For plate B, 18 degrees at the oblate and 72 degrees at the acute. Amazingly, plates of the same type so cut will only assemble into the fat and skinny three-dimensional cells that are the basic building blocks of a three-dimensional quasicrystal. Here again there is an economy that speaks to the deep mathematical structure of quasicrystals: the patterns of the bevels are exactly the pattern of the well-known local matching rules for the two-dimensional quasicrystal, the Penrose pattern (alas, not fool-proof rules.)

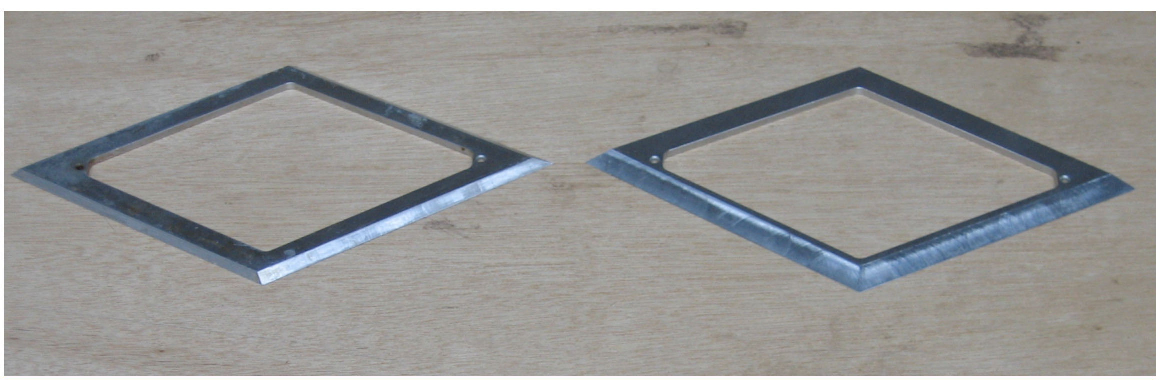
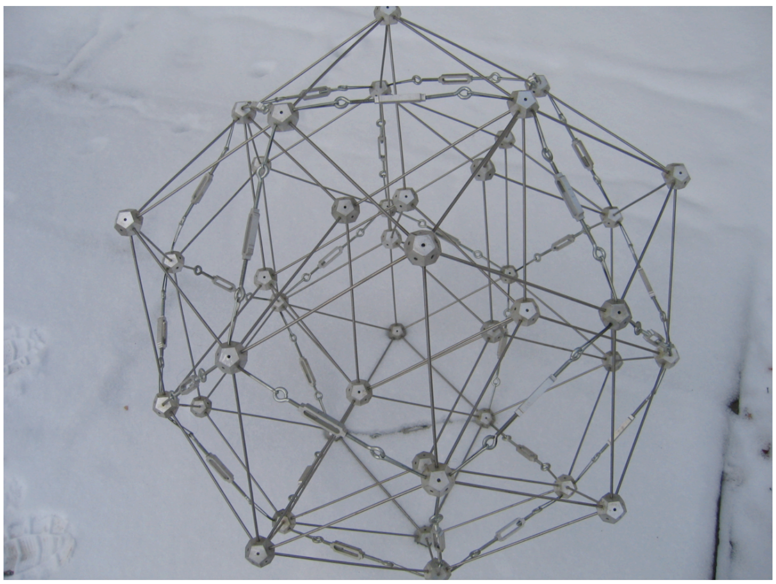


Plate A is on the left. Plate B is on the right.

Consider again the quasicrystal ball with turnbuckles. As mentioned, the triacontrahederal hull is derived from a dodecahedron and its dual an icosahedron: the twenty vertices of

the dodecahedron and the ten vertices of the icosahedron are kept, the edges (they bisect in this scaling) are discarded, and then all the thirty vertices are connected by new edges of equal length. In the photo below, one can see that the turnbuckles re-establish the dodecahedron with its pentagonal sides. These pentagons can neither deform nor rotate due to neighboring turnbuckles. And therefore, the quasicrystal ball is functionally a plate structure. As Wester has repeated reminded us in print, a plate-structure dodecahedron is stable because three pentagonal plates meet at each
corner.



Philosophical considerations

The 2D Penrose pattern is a special case of a three-dimensional quasicrystal: the case when the cells are turned so that one set of members is completely foreshortened to non-existence. But as we have seen with the 3D bevels being identical to the 2D matching rules, such a projection retains the information of the higher dimensional version, just as quasicrystals in general retain the information of the cubic lattices when projected. This is precisely the information needed to understand their optimal mathematical and physical structure. Further, the general insight that the projected figures retain essential information from higher-dimensional regular grids is the secret of their mystery. Against all intuition, quasicrystals retain their perfect tessellation, their uniformity of edges, uniformity of node and node orientation, and their long-range orientation that was part of their cubic, pre-projected state. Uniformity of parts makes them ideal candidates for structures; special projection generates their visual richness. Deep inside the algorithm of their construction are the secretes of the rigidity, and thus the path to their use in architectural structures.

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Francis and students?

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