

# The Continuous Droste Effect

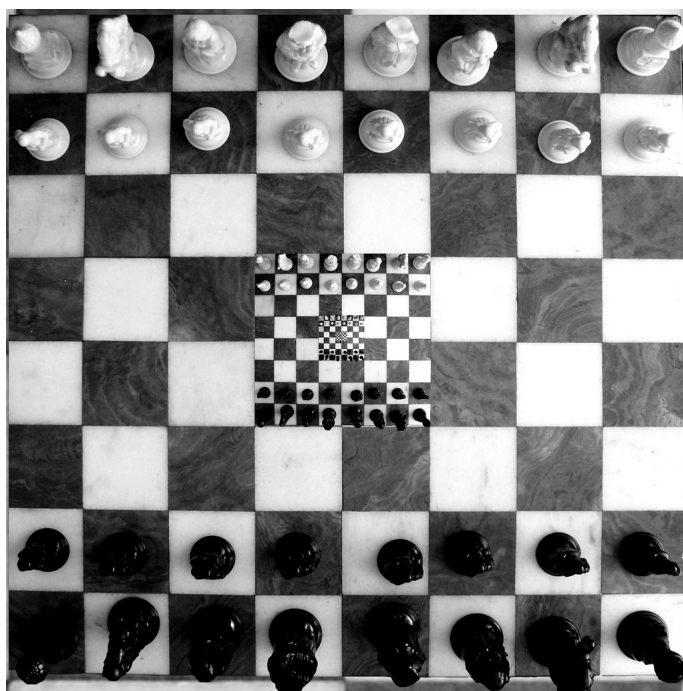
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The Droste Effect is a graphical effect where a picture is defined recursively in terms of itself. It was named after Droste, a cocoa maker which used this in its packaging.

In this way, one form of the Droste Effect is to leave a space in the picture where you wish to copy a scaled down version of the whole picture. Of course, this scaled down version will also have a scaled down space, which you fill recursively. In theory this process would go on infinitely, but practically it is truncated by the resolution of the image, or of your eye.

In most cases, one is only able to see only a few instances of the original picture clearly due to the picture scaling down geometrically.



(A discrete Droste picture from user FJTU's flickr set.)

Another form of Droste effect, and the one which gives a more transformed result is when one applies an exponential spiral to the picture (more about this later), in effect connecting the instances of the picture continuously. Thus instead of travelling around one instance in a circle, one can travel around in a spiral.

One can see the original picture on the left and the result of the continuous effect on the right.

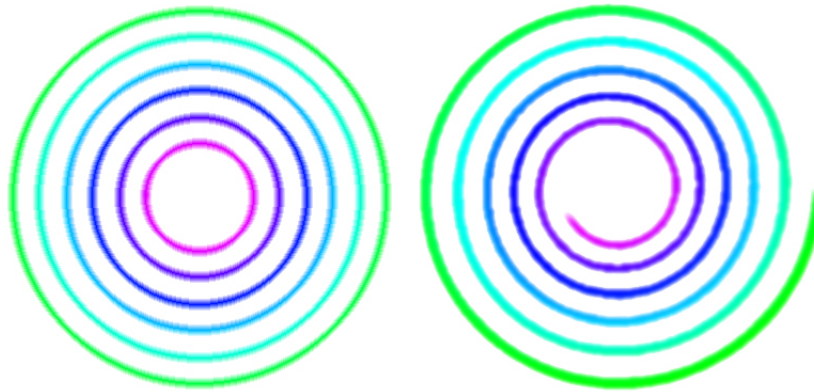


Note that the top example uses one spiral, and the bottom example uses two spirals, but the idea is still the same. There are several ways to generalize the basic case, such as using more than one spiral, or warping the spiral in some direction.

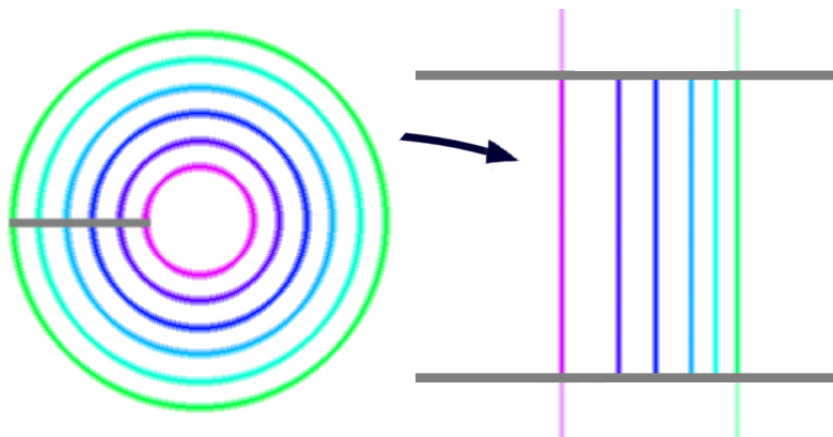
Jos Leys gives a fantastic explanation of the Droste Effect at the website [http://www.josleys.com/article\\_show.php?id=82](http://www.josleys.com/article_show.php?id=82) (accessible 4/2/2010.) which is great for explaining the basic method. I will paraphrase the method used, and a few ways to branch out from the basic method to become more creative.

The most basic example of this "continuous droste effect" is replacing concentric circles with a spiral. Thus, to start, the image should have a circular frame. One can see that if one replaced the inside of the circle with the annulus outside the circle in a recursive fashion, the discrete droste effect would be formed. In effect, the picture would consist of an infinite number of annuli inside each other, so that the border then would form several concentric circles.

So if one could continuously match up the left side of one annulus with the right side of the annulus inside, one could create a continuous spiral like that on the right. (Note how the colors remain the same in the circles on the left, but slowly change as one goes around the spiral on the right.)



In order to match up the sides of the annulus, it is easier to transform the annulus into a rectangle, work on the rectangle, and then transform back at the end. The logarithm map is useful for mapping the working annulus into a rectangle. Since the exponential map is  $2\pi$  periodic, we can simply work with a branch of the logarithm whose image results in a rectangle side by side of height  $2\pi$ . Once this rectangle has been obtained, it is repeated left and right. (This is what causes the recursive filling in for the discrete Droste effect.)

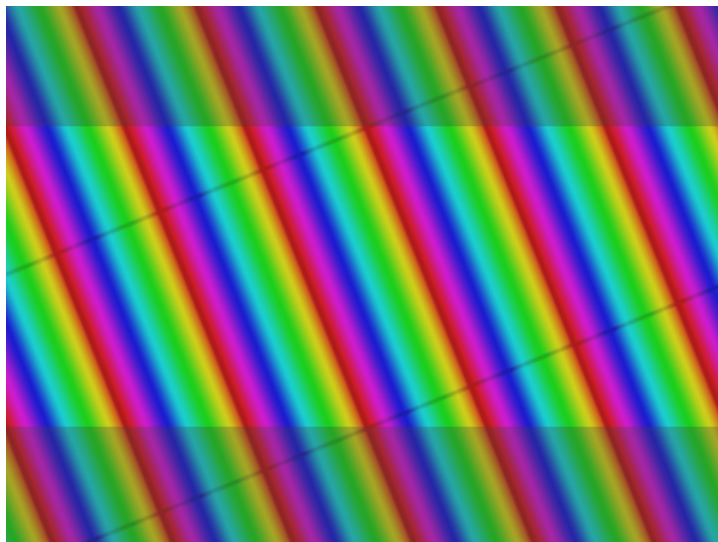


Note that if we took the exponential map of these rectangles, the top and bottom of each one would match up. However, to create the spiral effect, we want the top of one rectangle to match up with the bottom of the rectangle on its left. Thus, the simple solution is to rotate the rectangle slightly before transforming it back using the exponential map.

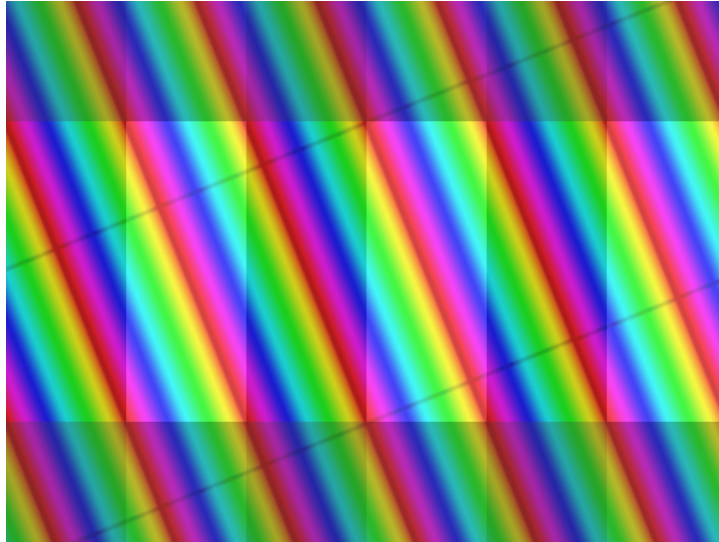
However note that the rectangle should actually be distorted to the left as a result of equally spaced concentric circles mapping to logarithmically spaced vertical lines in the exponential map. So one aspect of this transformation which is not clear to me is how this rotational step affects this distortion.



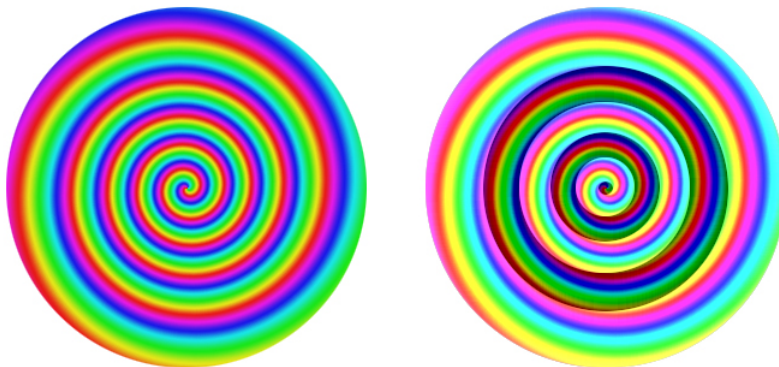
What is clear is that this sort of rotation will produce the desired effect in some way by connecting the top of rectangle with the bottom of the next. The picture below offers one way to visualize this – if we simplify the image and create multiple copies of the image (under the logarithm map) and stacks them as layers on top of each other, one can follow the stripes up and slightly to the left (these represent concentric circles in the original picture). The stripes are continuous between the vertical layers and thus it's apparent the top and bottom of the will "match up" so there is no seam when the vertical lines are once again mapped to concentric circles.



Due to an optical illusion, it may seem like the top and bottom don't actually match up, so here is another picture, which emphasizes the matching up. Since the stripes now slant left as they move up, under the exponential map the stripes will turn clockwise as they move in. One can contrast this with the original picture, in which the stripes merely move up and down. (Note that the panels below are rectangles, though they look distorted.)

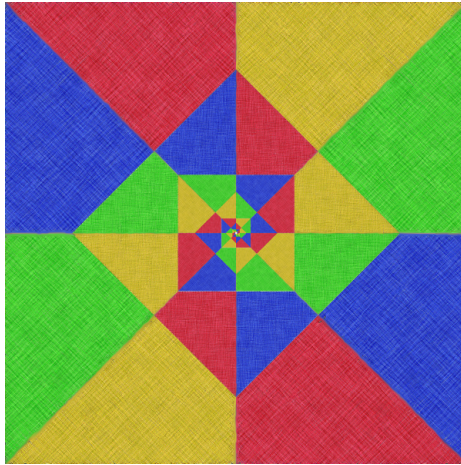


Taking an exponential of this diagram gives the desired spiral. On the left is given the original spiral, and on the right the highlighting of the specific sections, which are each self-similar. If the colors were simply rotating around the center point rather than swirling in, this would correspond to a discrete Droste effect.

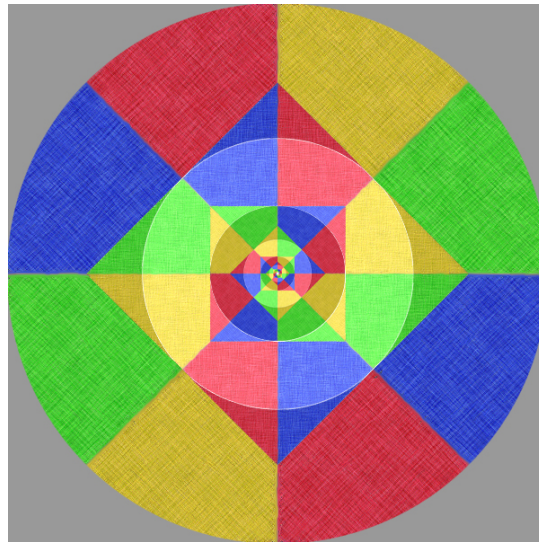




Putting all three steps together (logarithm, rotate, and exponential) as a composition gives the resulting continuous Droste effect as opposed to the discrete case. Once one has this framework, one can become more creative; using an "irregular annulus", such as a square with a square inside cut out, or even an irregular shape with a similar irregular hole cut out inside.



One way to see this is to simply reduce it to the previous case of a circle. One can draw a circle which touches the figure at the point closest to the center, then move/rescale the part outside the circle to the inside space of the annulus. This is most easily seen with a square, but it works for any shape. (This will still work for "annuli" of small width, though the process may need to be followed more than once.)



One can generalize this process even further to shapes where the center of the outer shape is different than of the inner shape (which is more common when continuing this process on photography) but I will direct the interested reader to either the previously mentioned website or the paper itself for details of this adaptation.

One implementation of the Droste Effect appears on the Wolfram Blog at <http://blog.wolfram.com/2009/04/24/droste-effect-with-mathematica/> (Accessible 4/2/2008). Here it discusses the Droste Effect with Mathematica and there is a fair amount of coding necessary to achieve the result, but the blog explains each the role of chunk of code and provides several examples of the end result.

Another implementation is in the Mathmap plugin for GIMP, a free imaging program. Although it requires two downloads (one for GIMP and one for Mathmap), both are free, and thus it makes an ideal program for creating one's own images. Mathmap requires the Droste implementation source code to work, but this and a great (and short) tutorial is available as a photo album at <http://www.flickr.com/photos/joshsommers/sets/72157594515046947/> (The tutorial states it is for advanced users, but selecting and cropping are pretty common tasks, and easily learned even if you are a beginner.)

In general the Droste Effect starts out from simple beginnings, but through adaptation and creativity one can create some surprising images. I invite any readers to look at the previously mentioned links and try it out with their own images, whether from the GIMP implementation or any others accessible. There are many different results from variations on this technique.