Latex Example 2

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Theorem. *For all real numbers x and y,*

$$|xy| = |x||y|$$

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Proof. To prove this, first suppose that $x \geq 0$ and $y \geq 0$. Then $xy \geq 0$. By definition of absolute value, $|xy| = xy$, $|x| = x$ and $|y| = y$. Therefore $|xy| = |x||y|$.

Next suppose that $x \geq 0$ and $y < 0$. Then $xy \leq 0$. By definition, $|xy| = -(xy)$, $|x| = x$ and $|y| = -y$. Since $-(xy) = x(-y)$, we conclude that $|xy| = |x||y|$.

Next suppose that $x < 0$ and $y \geq 0$. The argument from the previous paragraph, with the roles of x and y reversed, shows that $|xy| = |x||y|$.

Finally, suppose that $x < 0$ and $y < 0$. Then $xy > 0$. By definition, $|xy| = xy$, $|x| = -x$ and $|y| = -y$. Since $xy = (-x)(-y)$, we conclude that $|xy| = |x||y|$.

Having considered all possible cases for the signs of x and y, we have proved that $|xy| = |x||y|$. q.e.d.

This proof is of importance because it establishes that absolute value commutes with multiplication, a commonly used property of absolute value. The proof illustrates the strategy of dividing a mathematical statement into several cases and proving each case separately. Once the statement was broken tiny bit of algebra. This demonstrates the importance of going back to definitions when constructing proofs.