CLASS M7

Note Title 2/25/2008

Content 3-4 pm   method 4-5 pm

pocket.pdf (dynamical systems)

$$f : X \times P \to X : (x,p) \to f_p(x)$$

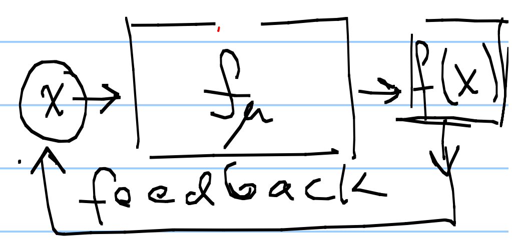
function f

Statespace X aka Phase Sp., Config. Sp.

parameter space P

Iterated function system IFT

$f_\mu,\ f_\mu^2 = f_\mu \circ f_\mu,\ f_\mu^3 = f_\mu^2 \circ f_\mu.$



example

$$f(z) = z^2 + \mu \qquad z, \mu \in \mathbb{C}$$

$$\text{Mandelbrot Set} = \{\mu \mid \lim_{n \to \infty} f_\mu^n(0) = \infty\}$$

Logistic Chaos (Allerton pocket r tica)

$f : [0,1] \times [0 \times 1] \to [0,1] : f_a(x) = 4a(1-x)x$

<u>Lorenz</u> (continuous dyn sys)

$\dot{X} = F(x)$ variants $F_p(x)$, $F(x,t)$

$dx = F(x)dt \rightsquigarrow f(x) = x + F(x)h$

f inherits the parameters of F plus h

<u>Gasket</u> (discrete dyn sys)

$f(x,y) = \left(\frac{x+a_i}{2}, \frac{y+b_i}{2}\right)$ $i = 1,2,3) = 1/3$

$(a_i, b_i) = (0,0), (0,1), (1,0)$ for example

$$\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1/2 & 0 & 1/2 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

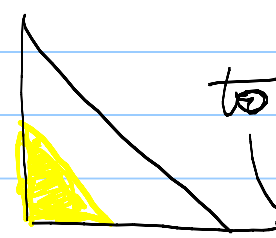
Homogeneous Coordinates (example 2)

$$\left[\begin{array}{cc|c} 1/2 & 0 & 1/2 \\ 0 & 1/2 & 0 \\ \hline 0 & 0 & 1 \end{array}\right]\left[\begin{array}{c} x \\ y \\ \hline 1 \end{array}\right] = \begin{array}{c} \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1/2 \\ 0 \end{bmatrix}[1] \\ [0\ 0]\begin{bmatrix} x \\ y \end{bmatrix} + 1 \cdot 1 \end{array} = \left[\begin{array}{c} \frac{x+1}{2} \\ \frac{y+0}{2} \\ \hline 1 \end{array}\right]$$

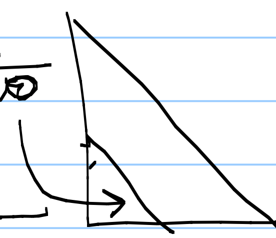
which expresses the Euclidean motion

$$\begin{bmatrix} x \\ y \end{bmatrix} \longrightarrow \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1/2 \\ 0 \end{bmatrix}$$ as matrix mult.

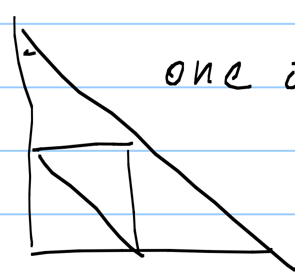
The contraction takes



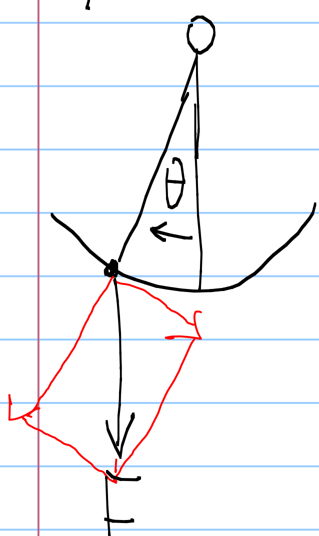
to



and the stochastic part (add $\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1/2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1/2 \end{bmatrix}$) translate it to one of 3, but <u>not</u> to the middle Δ.



pendulum — "Galileo's pulse"



first, resolve the force vector into (radial, tangential) components $(F\cos\ ,\ F\sin\theta)$

so radial force is always balanced by the tension in the string

Tangential equation

$$\ddot{\theta} = -\sin\theta \;\rightsquigarrow\; \ddot{x} = -x \;\rightsquigarrow\; \begin{cases} \dot{x} = y \\ \dot{y} = -x \end{cases}$$

pendulum   harmonic DE

Solution is $(x, y) = (\sin t, \cos t)$ but

```
      x = x0, y = y0
  ┌─> xold = x
draw  x = x + y*h
  │   y = y - xold*h
  └── x = xold
```

this lies

but

```
x = x0, y = y0
┌─> x = x + y*h
│   y = y - x*h
└── ...
```

this closes up.

Why?

Comment

There are several methods for "solving" a continuous dynamical system discretely. For the harmonic DE $\ddot{x} = -x$, Euler's method is good in that it is simple, its bad because none of its orbits (= curve traced out by a particle) correspond to the orbits of the cont dyn sys

The method of Gauss - Seidel is better because, at least, the orbits are closed ovals even though still not circles.

For the Lorenz, even Euler gives good results for visualizing the attractor.

Exercise

Given the algorithm $\left[\begin{array}{l} x = x + y*h \\ y = y - x*h \\ \text{draw} \end{array}\right.$ determine the IFT which effected by this.

NB There was a brief discussion of the Mandelbrot & Julia examples but well redo that some other time.