Modeling Wind Turbines and Vortices Using VPython

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1 Abstract

Recent investigations of the power output of wind turbines have found that the addition of winglets to the tips of turbine blades can increase the power output of the turbine. This effect is largely achieved by the reduction of tip loss effects and the translation of wing tip vortices away from the turbine blade.

This project aims to model wind turbines with winglets and the vortices that occur at the tips of the turbine blades. Using the graphics module VPython, these wind turbines and vortices can be modeled at variable flow rates (wind speeds).

2 Background

The wind turbine has been used as a method of extracting power or accomplishing auxiliary tasks for quite some time. Windmills, for instance, have been used to draw water from wells or to run grain mills. The wind turbines that this project aims to model, however, are those used for extracting kinetic energy from wind and converting it to electrical energy. The following sections detail the governing forces and phenomena surrounding wind turbines.

2.1 Turbulent Flow

When relative motion occurs between a fluid and a solid, various forces occur at the interface of these two "objects." Many of these forces can be reduced to two categories: inertial forces, which include forces such as lift, drag, and simple momentum transfer from the fluid to the object in the fluid, and viscous forces, which is the fluid equivalent of friction that occurs along the interface between the fluid and the object impeding its motion. When examining wind turbines, the associated flows are referred to as *turbulent*. What this means is that the *Reynolds number*, or the dimensionless ratio of *inertial* forces, which generally include a transfer of momentum to *viscous* forces, which are the equivalent of friction for fluids, is especially high—on the order of several hundred thousand for external flow—and that the viscous forces can largely be neglected [1]. The Reynolds number is given by:

$$R_e = \frac{vD}{\nu} \tag{1}$$

where v is the velocity of the fluid, D is some characteristic length scale, such as the chord length of a wing, and ν is the *kinematic viscosity* of the fluid, which is a property that defines a fluids resistance to relative motion. The numerator of this equation defines the inertial effects, while the denominator defines the viscous effects.

2.2 Aerodynamic Forces

In the context of wind turbines, the blades are very similar to the wings of an airplane, and the forces that act on each of these objects as they move through the fluid are very similar. Lift, which is the reaction force acting on the blade or wing as a result of pushing the fluid downward, acts vertically and perpendicular to the chord of the airfoil when looking at the wing's cross section. Drag acts parallel to the chord of the airfoil, and is the reaction force acting on the wing in response to displacing the fluid horizontally. The *no-slip condition* states that at the interface of any two fluids, or an object and a fluid, there can be no relative motion. Because of this, there are viscous forces that occur at the interface between the wing and the air that produce the resultant drag. These are the two primary aerodynamic forces acting on the wing and the ones with which we are most concerned.

As the wing moves through the fluid, a *pressure gradient*, or difference in pressure, is created from the bottom of the wing to the top of the wing. Because of this, the fluid flows over the wing in a chord-wise direction, but it also flows from the root, where the wing or blade connects with the fuselage or turbine axis, down the span of the wing and up and around the tip. At the tip of the wing, where the span-wise and chord-wise flows "collide," wing-tip vortices are formed.

When these vortices form, an induced drag is produced on the wing or blade. What this means is that the effective angle of attack, the angle that the chord line forms with the horizontal, is reduced, requiring the wing to tilt further backward to compensate for this loss; however, as the wing tilts further back, the drag acting on the wing increases, as the projected area of the wing in the fluid's flow increases. The equations for lift and drag are as follows:

$$L = \frac{1}{2}\rho v^2 A C_L \tag{2}$$

$$D = \frac{1}{2}\rho v^2 C_D A \tag{3}$$

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[1] Bryngelson, Spencer. ME 310: Fluid Mechanics Lecture, October 16, 2015.

where A represents the planar area of the wing when referencing lift but represents the cross-sectional area of the wing impeding the flow when referencing drag.

2.3 Addition of Winglets

In the case of a plane, this effect is not entirely disastrous, as the angle of attack can be modified at will. In case of a turbine, on the other hand, the angle of attack is constant, and thus the decrease in effective angle of attack is something for which the turbine cannot compensate. Instead, to reduce the effects of the vortices, winglets are installed at the tips of the turbine blades. These winglets increase the effective *aspect ratio*, the ratio of the plane's wing span to its chord length, without actually lengthening the wing. This is optimal for turbine blades and wings alike, as design limitations prevent the indefinite lengthening of wings and blades. An increase in aspect ratio reduces the coefficient of drag, as the coefficient of induced drag is inversely proportional to the aspect ratio, as seen in equation one, where C_L is the coefficient of lift, AR is the aspect ratio, and e is the wing span efficiency number, as determined by the geometry of the wing[2].

$$\mathrm{C}_{Di} = \frac{C_L^2}{\pi ARe} \tag{4}$$

By reducing the effective drag on the turbine blade, the wind turbine is then able to extract more power from the wind. There are some design considerations and side effects involved, as the addition of winglets increase the moment, the lift acting on the winglet multiplied by the lever arm, acting about the root of the turbine blade, but the increase in the coefficient of power is quite encouraging.

2.4 Vortex Generation

The vorticity of a fluid is defined mathematically as the cross product of the gradient vector with the velocity vector. That is:

$$\zeta = \vec{\nabla} \times \vec{v} \tag{5}$$

where ζ is the vorticity, $\vec{\nabla}$ is the vector containing the partial derivatives with respect to x, y and z, and $\vec{v}$ is the velocity vector containing the x, y, and z components of velocity. This equation provides us with the vector field of velocities of the fluid at a given point in space. For the purposes of modeling, vortices will be demonstrated on a particle-by-particle basis, or simply by referencing the general area in which they occur. In future expansions of this project, it would be interesting to model the entire velocity and vorticity fields in a differential analysis of the flow. Unfortunately, skill limitations prevented the development of the project to this extent.

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[2] "The Drag Coefficient." The Drag Coefficient. Ed. Nancy Hall. NASA, 5 May 2015. Web. 10 Oct. 2015.

3 RTICAs

3.1 Software Requirements

For the visualization portion of this project, VPython will be used. It allows for the simple creation of three dimensional objects, as well as the implementation of user input to control the behavior of the two RTICAs. VPython and Python 2.7 are the only software and module specifications necessary to execute these programs.

3.2 Modeling the Turbine

The file ”vpython_turbine.py” models a wind turbine whose rotor is in a plane normal to the flow of the air, or the wind. It shows the rotation of this turbine, as well as the generation of the vortices at the tips of the blades and the paths in which they propagate. Making use of the the ”frame” system of animation, the turbine itself is modeled, while the vortical paths are created with the ”curve” object as they trail behind the turbine.

The turbine has three main levels of frames–the world level, the blade level, and the winglet level. Each of the blades exists in its own blade frame, which is positioned relative to the world frame, and each of the winglets exists in its own winglet frame, which is positioned relative to the associated blade frame. The turbine tower and the rotor ”base,” represented with a cylinder, are each created in the world frame.

The turbine wake, which details the propagation paths and points of the wing tip vortices, is represented by three curves, each originating at one of the three winglets. Each curve is limited so that it can retain only 1000 points to prevents the over-expansion of the curve.

The ”while” loop at the base of the script governs all of animation and key events during the execution. With each pass through the loop, each blade is rotated by an angle that the user controls by increasing or decreasing the ”increment” variable. This is equivalent to the wind speed. By pressing the ’a’ or ’d’ key, the angle can be increased or decreased to a maximum of 8 pi radians or a minimum of 0 radians (stationary). Additionally, with each pass through the loop, a point is appended to each of the wake curves at the current location of the winglets. For the purposes of this RTICA, the absolute position of the winglets in the world frame is used.

Though the entire turbine appears to remain stationary while the wake trails behind it, the entire turbine moves forward in the ”z” direction by a value of increment. The center of the scene simply moves forward as well to create the illusion that the turbine is stationary.

3.3 Demonstrating Forces

The file ”forces.py” is the second RTICA, and it models a wing or blade and demonstrates the aerodynamic forces acting on the blade, as well as the paths that particles would take over the blade. The script allows for the control of several of the blade parameters, namely the angle of attack, the length of the blade, and the velocity of the wind. The arrows representing each of the forces then change as these parameters are modified.

For the animation of this script, only two frames are employed. These frames are the world frame and the frame of the blade, which is defined relative to the world frame. The blade itself is created by extruding the profile of an airfoil to the full length of the wing along the path defined by the variable *straight*. The coordinates of each point are taken from the data collected by UIUC Applied Aerodynamics group[3]

Before entering the "while" loop, several objects and values must be initialized. Objects that must be initialized are the three arrows representing the lift, drag, and lift-induced drag, as well as six sets of ten points that represent the air particles traveling over the wing. The values that must be initialized are the coefficients of lift, drag, and induced drag, as well as the velocity, angle of attack, and the length of the wing. These values are all used to calculate the magnitudes of each of the forces, which determine the lengths of the arrows that represent them. Finally, the key handling function is defined, which allows the user to control the angle of attack ('d' to increase, 'a' to decrease), the velocity ('w' to increase, 's' to decrease), and the length ('o' to increase and 'l' to decrease).

As in "vpython_turbine.py," a while loop at the base of the script controls the animations that occur. Additionally, it dynamically calculates values of each of the forces and updates the arrows as each of the parameters is varied. Six "for" loops–one for each "streamline"–continually update the position of each of the points to demonstrate the motion of the particles. The four interior sets simply pass over the shape of the wing, while each set of points on the edges of the wing follow the vortical path as they travel over the wing.

4 Conclusion

Through the use of VPython, the interaction of wind with the wind turbine was simulated. There is still work to be done in refining these RTICAs, though they effectively demonstrate vortices and aerodynamic forces. Future additions to the current project would include making each RTICA more aesthetically pleasing, as well as more accurately representing the vortices, and having them dynamically respond to parameter changes. In a continuation, a third RTICA would likely be created to demonstrate a differential analysis of the flow in the form of vector fields for velocity, pressure, and vorticity.

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[3] "UIUC Airfoil Data Site." UIUC Airfoil Coordinates Database. UIUC Applied Aerodynamics Group, 2015. Web. 10 Nov. 2015.

5 References

1. Bryngelson, Spencer. ME 310: Fluid Mechanics Lecture, October 16, 2015.
2. "The Drag Coefficient." The Drag Coefficient. Ed. Nancy Hall. NASA, 5 May 2015. Web. 10 Oct. 2015.
3. "UIUC Airfoil Data Site." UIUC Airfoil Coordinates Database. UIUC Applied Aerodynamics Group, 2015. Web. 10 Nov. 2015.