Modeling Wind Turbines and Vortices Using PyOpenGL

Charles Tierney

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1 Abstract

Recent investigations of the power output of wind turbines have found that the addition of winglets to the tips of turbine blades can increase the power output of the turbine. This effect is largely achieved by the reduction of tip loss effects and the translation of wing tip vortices away from the turbine blade.

This project aims to model wind turbines with winglets and the vortices that occur at the tips of the turbine blades. Using the graphics module PyOpenGL, the associated OpenGL extension GLUT, and/or the module PyGame, these wind turbines and vortices can be modeled at variable flow rates (wind speeds).

2 Background

The wind turbine has been used as a method of extracting power or accomplishing auxiliary tasks for quite some time. Windmills, for instance, have been used to draw water from wells or to run grain mills. The wind turbines that this project aims to model, however, are those used for extracting kinetic energy from wind and converting it to electrical energy. The following sections detail the governing forces and phenomena surrounding wind turbines.

2.1 Turbulent Flow

When relative motion occurs between a fluid and a solid, various forces occur at the interface of these two "objects." Many of these forces can be reduced to two categories: inertial forces, which include forces such as lift, drag, and simple momentum transfer from the fluid to the object in the fluid, and viscous forces, which is the fluid equivalent of friction that occurs along the interface between the fluid and the object impeding its motion. When examining wind turbines, the associated flows are referred to as "turbulent." What this means is that the Reynolds number (the dimensionless ratio of inertial forces to viscous forces) is especially high—on the order of several thousand—and that the viscous forces can largely be neglected[1].

2.2 Aerodynamic Forces

In the context of wind turbines, the blades are very similar to the wings of an airplane, and the forces that act on each of these objects as they move through the fluid are very similar. Lift, which is the reaction force acting on the blade or wing as a result of pushing the fluid downward, acts vertically and perpendicular to the chord of the airfoil when looking at the wing's cross section. Drag acts parallel to the chord of the airfoil, and is the reaction force acting on the wing in response to displacing the fluid horizontally. These are the two primary aerodynamic forces acting on the wing and the ones with which we are most concerned.

As the wing moves through the fluid, a pressure gradient, or difference in pressure, is created from the bottom of the wing to the top of the wing. Because of this, the fluid flows over the wing in a chord-wise direction, but it also flows from the root, where the wing or blade connects with the fuselage or turbine axis, down the span of the wing and up and around the tip. At the tip of the wing, where the span-wise and chord-wise flows "collide," wing-tip vortices are formed.

When these vortices form, an induced drag is produced on the wing or blade. What this means is that the effective angle of attack, the angle that the chord line forms with the horizontal, is reduced, requiring the wing to tilt further backward to compensate for this loss; however, as the wing tilts further back, the drag acting on the wing increases, as the projected area of the wing in the fluid's flow increases.

2.3 Addition of Winglets

In the case of a plane, this effect is not entirely disastrous, as the angle of attack can be modified at will. In case of a turbine, on the other hand, the angle of attack is constant, and thus the decrease in effective angle of attack is something for which the turbine cannot compensate. Instead, to reduce the effects of the vortices, winglets are installed at the tips of the turbine blades. These winglets increase the effective aspect ratio, the ratio of the plane's wing span to its chord length, without actually lengthening the wing. This is optimal for turbine blades and wings alike, as design limitations prevent the indefinite lengthening of wings and blades. An increase in aspect ratio reduces the coefficient of drag, as the coefficient of induced drag is inversely proportional to the aspect ratio, as seen in equation one, where C_l is the coefficient of lift, AR is the aspect ratio, and e is the wing span efficiency number, as determined by the geometry of the wing[2].

$$\mathrm{C}_d = \frac{C_l^2}{\pi ARe} \tag{1}$$

[1]Bryngelson, Spencer. ME 310: Fluid Mechanics Lecture, October 16, 2015.
[2]"The Drag Coefficient" Glenn Research Center, NASA.

By reducing the effective drag on the turbine blade, the wind turbine is then able to extract more power from the wind. There are some design considerations and side effects involved, as the addition of winglets increase the moment, the lift acting on the winglet multiplied by the lever arm, acting about the root of the turbine blade, but the increase in the coefficient of power is quite encouraging.

2.4 Vortex Generation

The vorticity of a fluid is defined mathematically as the cross product of the gradient vector with the velocity vector. That is:

$$\zeta = \vec{\nabla} \times \vec{v} \tag{2}$$

where ζ is the vorticity, $\vec{\nabla}$ is the vector containing the partial derivatives with respect to x, y and z, and $\vec{v}$ is the velocity vector containing the x, y, and z components of velocity. This equation provides us with the vector field of velocities of the fluid at a given point in space. For the purposes of modeling, the vorticity will likely be generalized to a two-dimensional velocity field, which will then follow a circular path in the wake of the wind turbine. These vortices will originate at the tips of the turbine blades, specifically at the tips of the winglets if possible.

3 PyOpenGL and GLUT/PyGame

For the visualization portion of this project, PyOpenGL and GLUT or PyGame will be used. PyOpenGL, in my brief studies of the module, handles the storage of edges and vertices in numerical arrays, while GLUT and PyGame handle the actual visualization of the object. I would prefer to use PyGame, as it has built in key handlers that will allow for the manipulation of perspective as well as potential for user input. It is very easy to create a rudimentary GUI as well, which can take this project to the next step.

4 Goals

My goals for the project are as follows:

1. Become proficient in LaTeX, OpenGL, and GLUT
2. Model wint turbines with vortices that originate at the tips of the blades
3. Allow for user control of perspective
4. Create an interactive method to examine the formation of vortices

Potential extensions of the project include:

1. Variable fluid flow rate

2. User input via JavaScript, and embedding the scripts into the web page.
3. Potential for vortices to be functions of the geometry of the blade.

5 Timeline

- October 30th: Create Stationary Turbine
- November 6th: Create turbine with rotating blades
- November 13th: Model Two dimensional vortices that follow circular profile around the blades' path
- November 20th: Add translation of vortices due to wind speed
- November 27th: Add variable flow rates/user interface