Mercator's Projection

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Introduction

- Cylindrical projection
- Derivation of equations
- Truncation and Scale Factor
- Loxodromes and Geodesics
- Calculating Distance

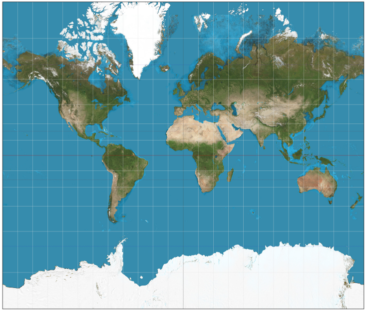


Figure 1 : Mercator's Projection

Projecting the Globe

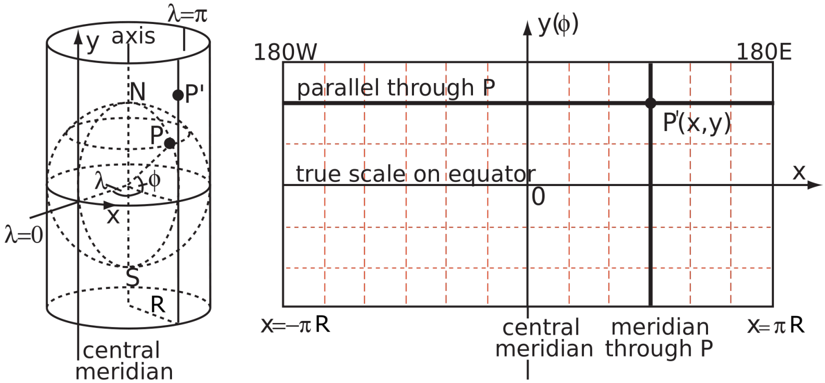
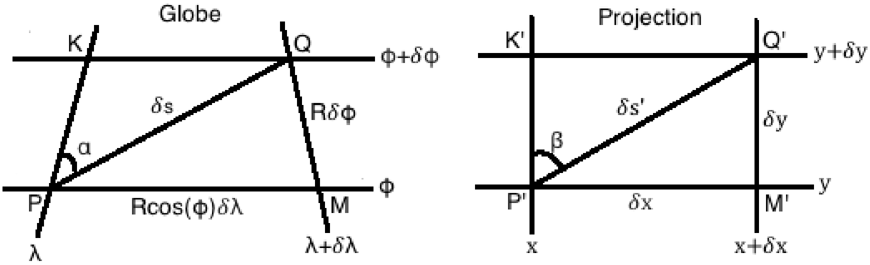


Figure 2 : Geometry of a cylindrical projection.

- Geographic coordinates of latitude ϕ and longitude λ
- Tangential to globe at equator
- Radius of a parallel is $Rcos(\phi)$

Small Element Geometry



$$tan\alpha = \frac{Rcos(\phi)\delta\lambda}{R\delta\phi} \text{ and } tan\beta = \frac{\delta x}{\delta y}$$

Parallel Scale Factor $k(\phi) = \frac{P'M'}{PM} = \frac{\delta x}{Rcos(\phi)\delta\lambda}$

Meridian Scale Factor $h(\phi) = \frac{P'K'}{PK} = \frac{\delta y}{R\delta\phi}$

Deriving Mercator's Projection

$$tan\beta = \frac{Rsec\phi}{y'(\phi)} tan\alpha,\ k = sec\phi,\ h = \frac{y'(\phi)}{R}$$

Equality of Angles: $\alpha = \beta \longrightarrow y'(\phi) = Rsec(\phi)$
Equality of Scale Factors: $h = k \longrightarrow y'(\phi) = Rsec(\phi)$

Therefore,

$$x = R(\lambda - \lambda_0) \text{ and } y = Rln[tan(\tfrac{\pi}{4} + \tfrac{\phi}{2})]$$

And inversely,

$$\lambda = \lambda_0 + \tfrac{x}{R} \text{ and } \phi = 2tan^{-1}[e^{(\frac{y}{R})}] - \tfrac{\pi}{2}$$

Truncation and Scale Factor

The ordinate y approaches infinity as the latitude approaches the poles.

$$\begin{aligned}\phi &= 2tan^{-1}[e^{(\frac{y}{R})}] - \frac{\pi}{2} \\ &= 2tan^{-1}[e^{\pi}] - \frac{\pi}{2} \\ &= 1.48842 \text{ radians} \\ &= 85.05133^\circ\end{aligned}$$

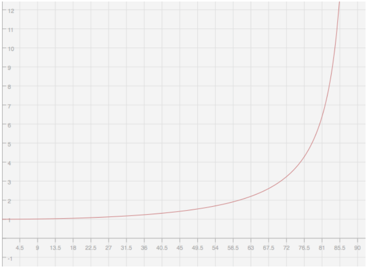


Figure 3 : Graph of Scale Factor vs. Latitude.

Loxodromes and Geodesics

Loxodromes

Paths, also known as rhumb lines, which cut a meridian on a given surface at any constant angle.
All straight lines on the Mercator's Projection are loxodromes.

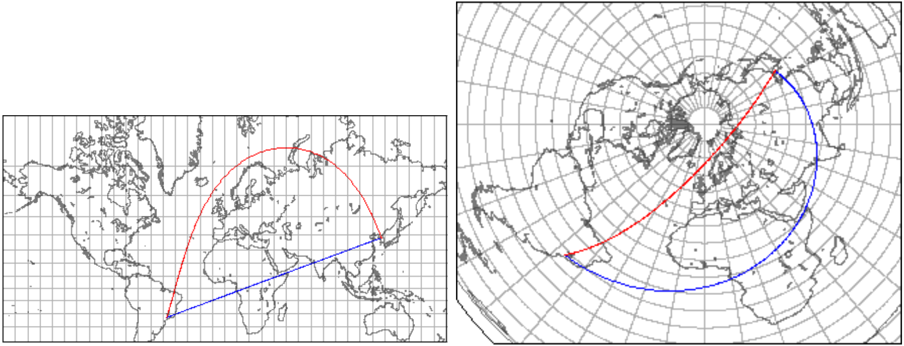


Figure 4 : Loxodromes vs Geodesics

Calculating Distance

Representative Fraction

The fraction $\frac{R}{a}$ is called the representative fraction.
It is also known the principal scale of the projection.
For example, if a map has an equatorial width of 31.4 cm, then its global radius is 5 cm, which translates to an RF of approximately $\frac{1}{130M}$.

There are two main problems when it comes to calculating distance using Mercator's projection:

- Variation of scale with latitude
- Straight lines on the map do not correspond to great circles

Short Distances: True Distance = rhumb distance $\cong$ ruler distance $\times \frac{cos\phi}{RF}$
For example, a line of 3mm, its midpoint at 40°, and an RF of $\frac{1}{130M}$, the true distance would be approximately 300km

Calculating Distance

Measuring longer distances requires different approaches

On the equator: True distance $= \frac{\textit{ruler distance}}{RF}$

On other parallels: Parallel distance = ruler distance $\times \frac{cos\phi}{RF}$

On a meridian: $m_{12} = a|\phi_1 - \phi_2|$

On a rhumb: $r_{12} = a \sec\alpha|\phi_1 - \phi_2| = a \sec\alpha\Delta\phi$

Sources

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