

1 Proposal

Due to my major in materials science, I wish to pursue something that is related to materials. I am particularly intrigued by the optimization and study of materials through the use of the natural (or created) defects in their crystal structures.

According to studies by H. Khanbareh, J. H. Kruhl, and M. Nega, one type of these defects, grain boundaries—the intersections of two crystals (grains) exists in fractal dimensions.^{1 2} Although grain boundaries are classified as 2-D defects, in aluminum alloys and in recrystallized quartz, their fractal dimensions lie between 1.05 and 1.30.³ In quartz, the dimensional range can be used to predict the temperature of the recrystallization, while in aluminum the range can be used to predict the fracture toughness of the alloy.⁴

Khanbareh, Kruhl, and Nega all note that the fractal nature of the studied materials is related to the Koch curve (Koch snowflake), which repeats according to the following pattern:

1. Take an equilateral triangle.
2. Split the sides into three equal lengths.
3. On any outward facing side (a side that is not connected to a previous triangle), take the central length and create an equilateral triangle from it.
4. For each line segment in the set, repeat from 1.⁵

As VPython is a 3-D modeling system, I do not feel that I would be utilizing its full power simply representing a Koch snowflake. I intend to use VPython to create (and somehow analyze) versions of the Koch curve that repeat from three-dimensional starting points. If one were to start with a tetrahedron, the pattern could be adapted as such:

1. Take a regular tetrahedron.
2. Find the centroid of each outer face.

¹Khanbareh, H. (2011, December). Fractal Dimension Analysis of Grain Boundaries of 7XXX Aluminum Alloys and Its Relationship to Fracture Toughness. Retrieved from http://www.lr.tudelft.nl/fileadmin/Faculteit/LR/Images/NovAM/Pictures_research/MSc_projects/Hamide_thesis.pdf

²Kruhl, J.H. & Nega, M. (1996). Geologische Rundschau, 85, 38-43. DOI:10.1007/BF00192058

³And probably lots of other materials!

⁴See 1, 2.

⁵Weisstein, Eric W. (2013). Koch Snowflake. Retrieved from <http://mathworld.wolfram.com/KochSnowflake.html>

3. Place a regular tetrahedron of side length equal to $1/3$ of the previous iterations such that the centroid of one its faces lies on the midpoint (2) and that its base corners lie on the medians of the equilateral triangle.
4. For each tetrahedron, repeat from 1.⁶

Hopefully, I will be able to create a program in VPython that repeats recursively according to this process.

2 Goals

I would like to have (at minimum) the following items completed by the end of the semester⁷:

1. Become proficient at VPython and LaTeX. (By end of semester. All submissions to be made in LaTeX henceforth)
2. Improve the aforementioned method for a tetrahedron fractal, so that it is a closer approximation of what a 3-D Koch curve would be. (By October 31, with a new LaTeX .pdf uploaded to my webpage)
3. Extend the idea of a 3-D Koch fractal to at least two other three-dimensional objects (presumably, spheres and cubes).⁸ (By mid-November)
4. Create a recursive VPython program that generates Koch-like fractals from the patterns in (3). (At least one by mid-November)
5. Learn how to and find approximations of the fractal dimensions of these objects. (Shortly after the completion of the VPython program(s))
6. Compare the fractal dimensions and properties of the generated fractals. (By end of semester)
7. Improve my webpage and post as many updates as possible. (Weekly)

⁶Sweeney, J. (2008, October). 3D Koch Snowflake. Retrieved from <http://www.3dvision.com/wordpress/2008/10/30/3d-koch-snowflake/>

⁷The order of completion will likely differ from the manner in which these items are listed.

⁸For an example of a spherical Koch curve, see Eric Haines' "sphereflake" <http://en.wikipedia.org/wiki/File:Sf6.jpg>