Fear The Sphere

David Prorok

December 16, 2013

Abstract

The goal of this project is to create a functioning game that can graphically display elastic collisions. The design is not to discover the unknown, but to demonstrate the known. The program takes a mostly dull concept and makes it interesting through the use of a video game facade.

1 The Game

The idea of the game is to put the user in control of a rubber ball. The user will earn coins as he or she progresses through a number of levels. The coins can then be used to increase the mass, speed, acceleration, etc. of his or her ball. The objective of each level is to knock a number of artificially intelligent rubber balls off a platform. The collisions will be realistic in that they will exhibit a conservation of momentum.

2 The Math

2.1 Collision Detection

As the program runs, it needs to determine whether or not two objects are in contact. It does this by testing the location of each ball against the other balls, and seeing if they are close enough to be touching. Mathematically, if

$$r_1 + r_2 \leq \sqrt{(C_{1x} - C_{2x})^2 + (C_{1y} - C_{2y})^2} \tag{1}$$

where
r_1 is the radius of the first ball
r_2 is the radius of the second ball
C_{1x} is the x-coordinate of the center of the first ball
C_{2y} is the y-coordinate of the center of the second ball, ect.,

then a calculation needs to be made (see Inelastic Collisions below). For the case where r_1+r_2 is less than the distance between the centers, an accomodation had to be made in order to prevent bugs such as objects getting stuck together, calculations being made wrong, etc. This was handled by taking the center of each ball and moving it until the equality of equation 1 becomes satisfied. They move in the direction that yields the shortest path.

$$C_{1x} = C_{1x} + \frac{(C_{1x} - C_{2x})}{|C_{1x} - C_{2x}|} \tag{2}$$

$$C_{1y} = C_{1y} + \frac{(C_{1y} - C_{2y})}{|C_{1y} - C_{2y}|} \tag{3}$$

Equations 2 and 3 work by either incrementing or decrementing the the coordinates of the ball a small amount until the edge of the ball is tangent to the edge of the ball it is colliding with. If C_{1x} is greater than C_{2x}, then C_1 will move to the right. If C_{1y} is less than C_{2y}, then C_1 will move upward, etc. (coordinate system is based off an origin in the top left corner of the environment).

2.2 Elastic Collisions

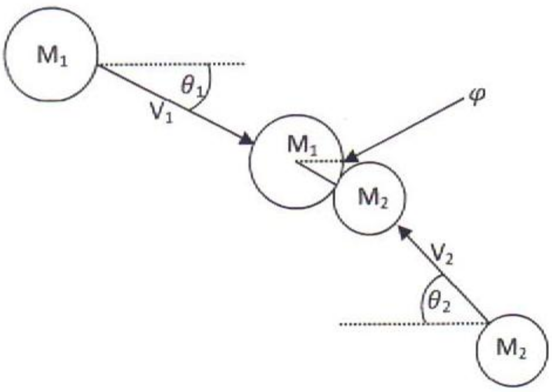

Figure 1: An example of a 2-dimensional collision. M_1 and M_2 travel from their top left and bottom right positions along the v_1 and v_2 vectors, respectively, before colliding at the location in the center.

A collision will occur whenever equation 1 is satisfied. Our goal is to determine the final velocities of each ball. In order to do this, we simplify the problem by rotating the axis so the collision looks like figure 2.

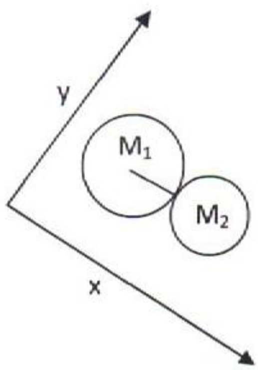

Figure 2: 2-dimensional collision after rotation of axis

In order to calculate the velocities relative to the rotated axis, we can use trigonometry. v_1, v_2, θ_1, θ_2, and ϕ are shown in figure 1. The subscript r stands for rotated.

$$v_{1xr} = v_1(cos(\theta_1 - \phi)) \tag{4}$$

$$v_{1yr} = v_1(sin(\theta_1 - \phi)) \tag{5}$$

$$v_{2xr} = v_2(cos(\theta_2 - \phi)) \tag{6}$$

$$v_{2yr} = v_2(sin(\theta_2 - \phi)) \tag{7}$$

The values of θ_1, θ_2, and ϕ can be determined using the coordinates of the center of each ball (C), the x- and y-velocities(pre-rotation) of each ball, and trigonometry.

$$\phi = arctan(\frac{(C_{1y} - C_{2y})}{(C_{1x} - C_{2x})}) \tag{8}$$

$$\theta_1 = arctan(\frac{v_{1y}}{v_{1x}}) \tag{9}$$

$$\theta_2 = arctan(\frac{v_{2y}}{v_{2x}}) \tag{10}$$

The axis are now rotated in such a way that the collision is only in the x_r direction. Since the collision is now 1-dimensional, we can use the conservation of momentum equation for a perfectly elastic collision:

$$v_{1f} = \frac{v_1(M_1 - M_2) + 2M_2v_2}{M_1 + M_2} \tag{11}$$

If you are unfamiliar with this equation, consider the following examples to gain some intuition: if $M_1 >> M_2$, v_1 will be unaffected; if $M_1 << M_2$, v_1 will become $-v_1$. By combining equations 4, 6 and 11, we get

$$v_{1xrf} = \frac{v_1(cos(\theta_1 - \phi))(M_1 - M_2) + 2M_2v_2(cos(\theta_2 - \phi))}{M_1 + M_2} \tag{12}$$

likewise,

$$v_{2xrf} = \frac{v_2(cos(\theta_2 - \phi))(M_2 - M_1) + 2M_1v_1(cos(\theta_1 - \phi))}{M_1 + M_2} \tag{13}$$

Now we must calculate the x- and y- velocities relative to the natural(non-rotated) axis. We do this using the contact angle, ϕ. We add $\frac{\pi}{2}$ when using the y-velocity because we are moving along the rotated y-axis and our ϕ is relative to the rotated x-axis.

$$v_{1xf} = v_{1xrf}cos(\phi) + v_{1yr}cos(\phi + \frac{\pi}{2}) \tag{14}$$

$$v_{1yf} = v_{1xrf}sin(\phi) + v_{1yr}sin(\phi + \frac{\pi}{2}) \tag{15}$$

$$v_{2xf} = v_{2xrf}cos(\phi) + v_{2yr}cos(\phi + \frac{\pi}{2}) \tag{16}$$

$$v_{2yf} = v_{2xrf}sin(\phi) + v_{2yr}sin(\phi + \frac{\pi}{2}) \tag{17}$$

We can now form our master equations. By plugging equations 5, 7, 12 and 13 into equations 14-17, we now have equations that can directly compute the final velocities.

$$v_{1xf} = \frac{v_1(cos(\theta_1 - \phi))(M_1 - M_2) + 2M_2 v_2(cos(\theta_2 - \phi))}{M_1 + M_2} cos(\phi) + v_1(sin(\theta_1 - \phi))cos(\phi + \frac{\pi}{2}) \tag{18}$$

$$v_{1yf} = \frac{v_1(cos(\theta_1 - \phi))(M_1 - M_2) + 2M_2 v_2(cos(\theta_2 - \phi))}{M_1 + M_2} sin(\phi) + v_1(sin(\theta_1 - \phi))sin(\phi + \frac{\pi}{2}) \tag{19}$$

$$v_{2xf} = \frac{v_2(cos(\theta_2 - \phi))(M_2 - M_1) + 2M_1 v_1(cos(\theta_1 - \phi))}{M_1 + M_2} cos(\phi) + v_2(sin(\theta_2 - \phi))cos(\phi + \frac{\pi}{2}) \tag{20}$$

$$v_{2yf} = \frac{v_2(cos(\theta_2 - \phi))(M_2 - M_1) + 2M_1 v_1(cos(\theta_1 - \phi))}{M_1 + M_2} sin(\phi) + v_2(sin(\theta_2 - \phi))sin(\phi + \frac{\pi}{2}) \tag{21}$$

These equations provide us with a "simple" way to calculate the final velocities of two colliding balls that very accurately mirrors reality.

3 References

Elastic Collisions. William Craver. URL: http://williamecraver.wix.com/elastic-equations [cited 11/20/2013].