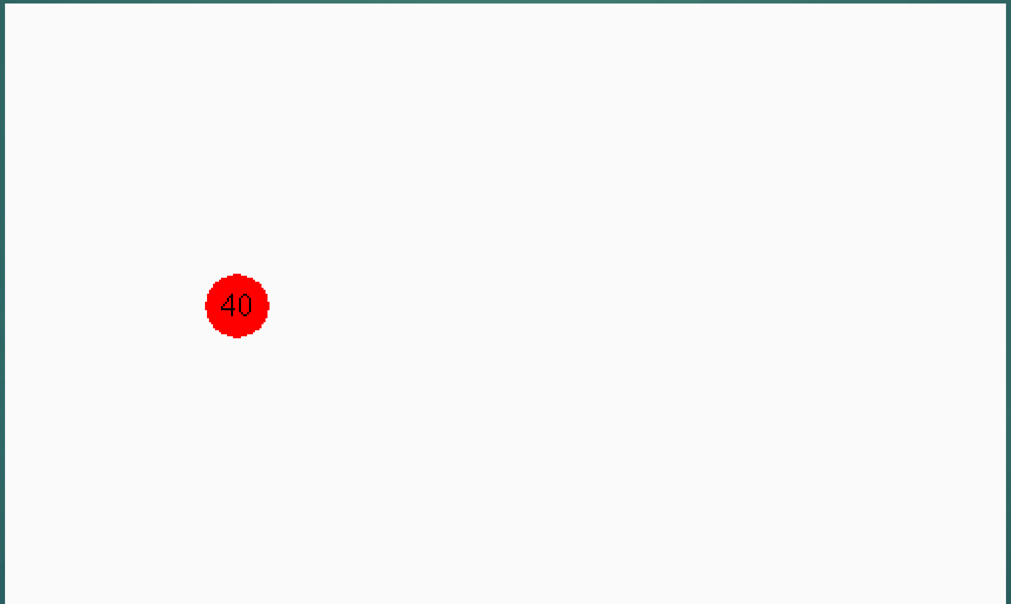
Fear The Sphere

TWO-DIMENSIONAL COLLISIONS BY DAVID PROROK

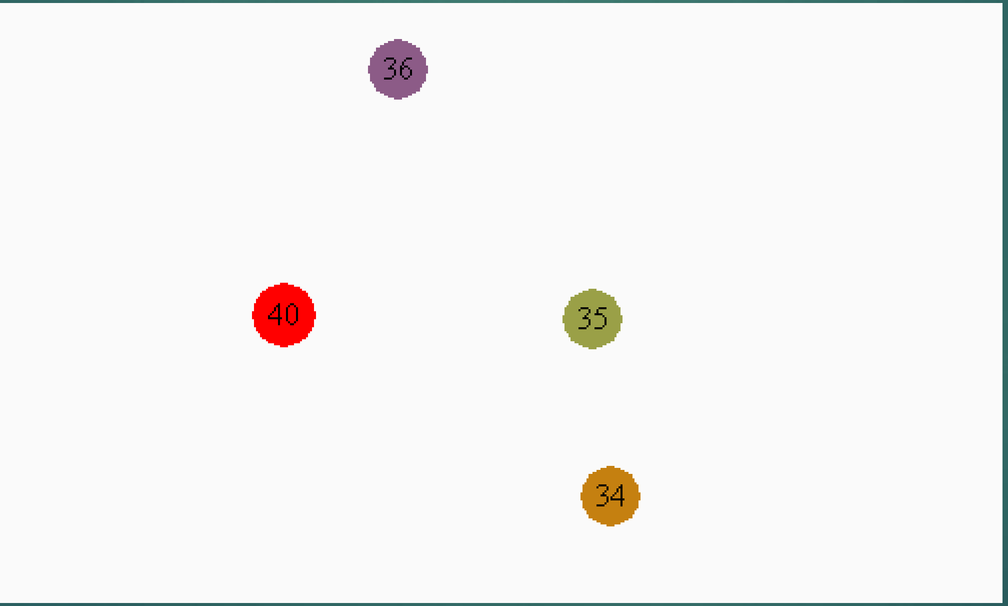
The Game

- Control a Ball



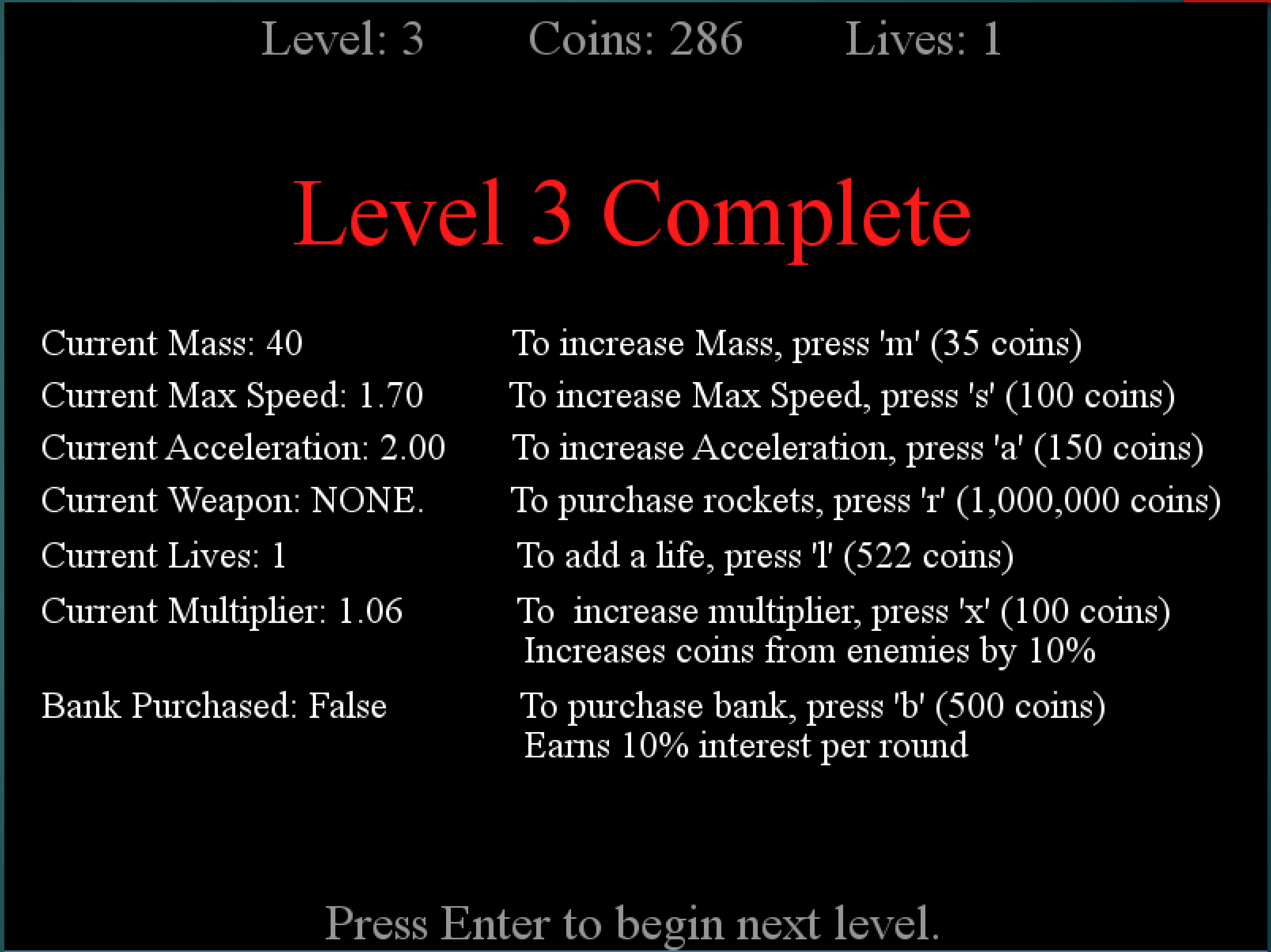
The Game

- Defeat Enemies



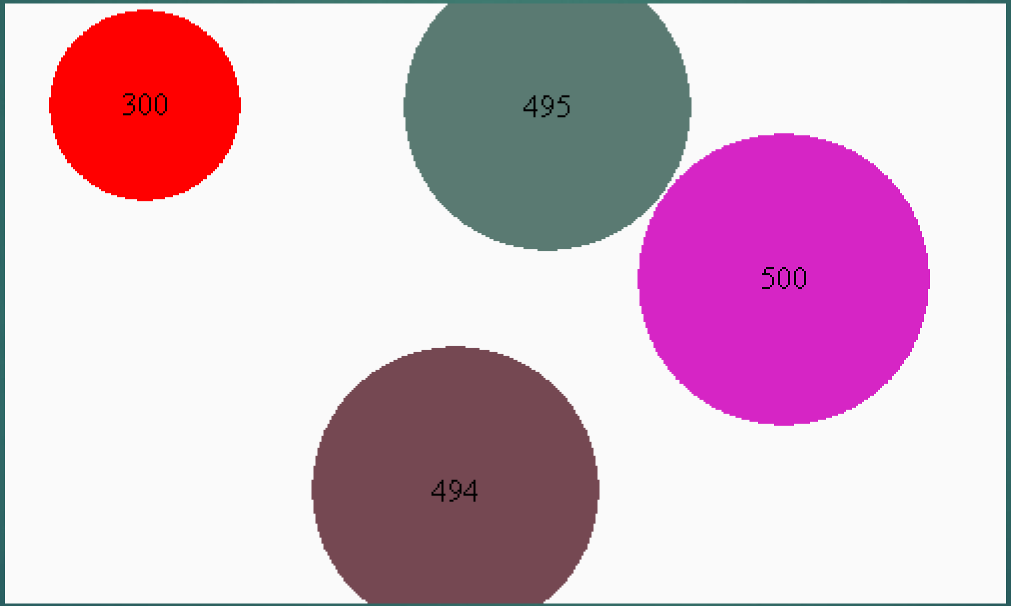
The Game

- Upgrade



The Game

- Repeat



The Math
Collision Detection

- The game needs to determine when two objects are colliding

$$r_1 + r_2 \leq \sqrt{(C_{1x} - C_{2x})^2 + (C_{1y} - C_{2y})^2}$$

where
r_1 is the radius of the first ball
r_2 is the radius of the second ball
C_{1x} is the x-coordinate of the center of the first ball
C_{2y} is the y-coordinate of the center of the second ball, ect.,

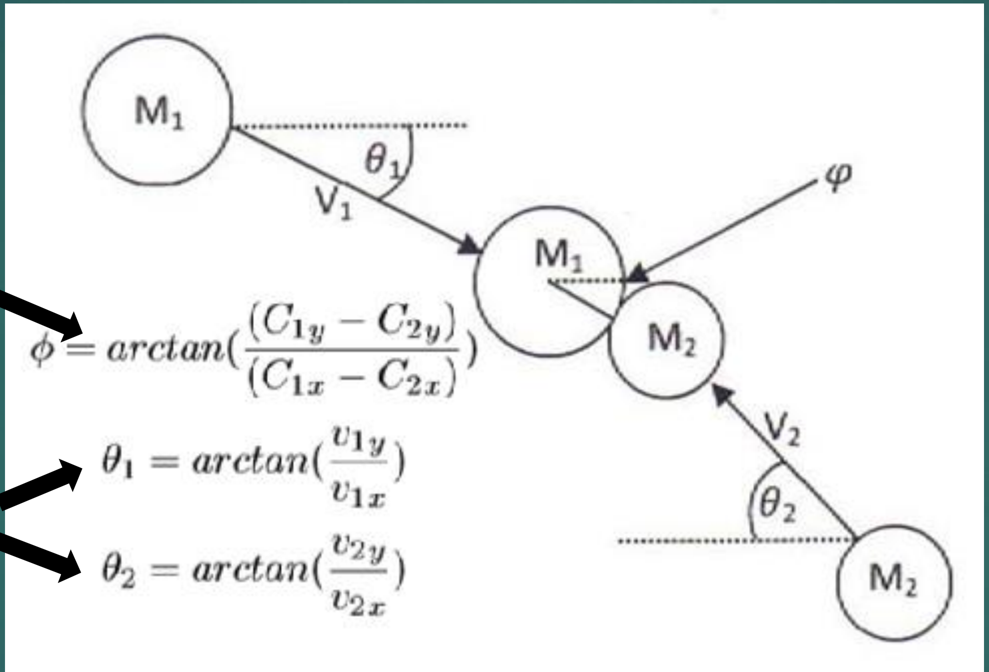
The Math
Collision Detection

- If two objects overlap, an adjustment needs to be made

$$C_{1x} = C_{1x} + \frac{(C_{1x} - C_{2x})}{|C_{1x} - C_{2x}|}$$

$$C_{1y} = C_{1y} + \frac{(C_{1y} - C_{2y})}{|C_{1y} - C_{2y}|}$$

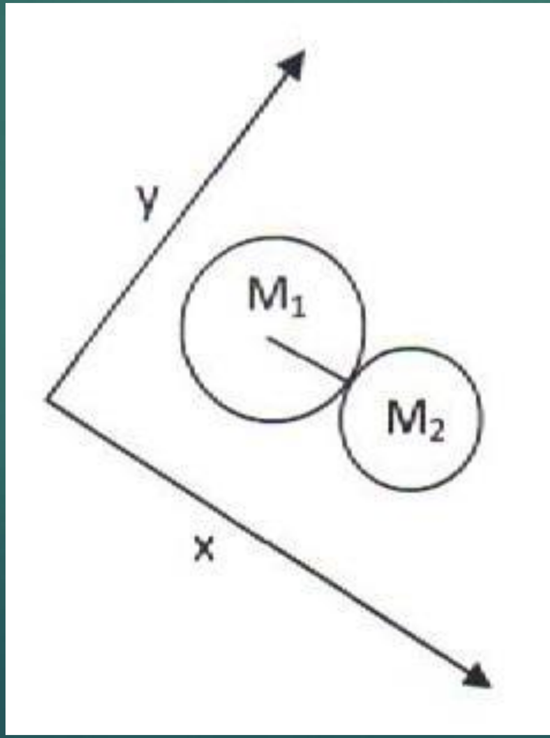
The Math
Elastic Collisions



- M1 and M2 travel from their top left and bottom right positions along the v1 and v2 vectors, respectively, before colliding at the location in the center

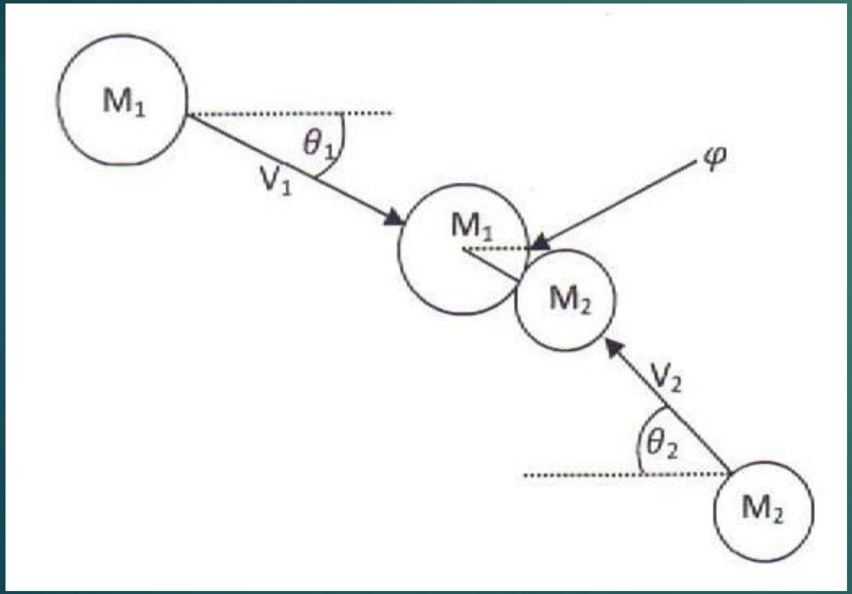
The Math
Elastic Collisions

- We need to calculate their final velocities
- We can simplify the problem by rotating the axes



The Math
Elastic Collisions

- In order to calculate the velocities relative to the rotated axis, we can use trigonometry. v1, v2, θ1, θ2, and φ are shown on the left. The subscript r stands for rotated.

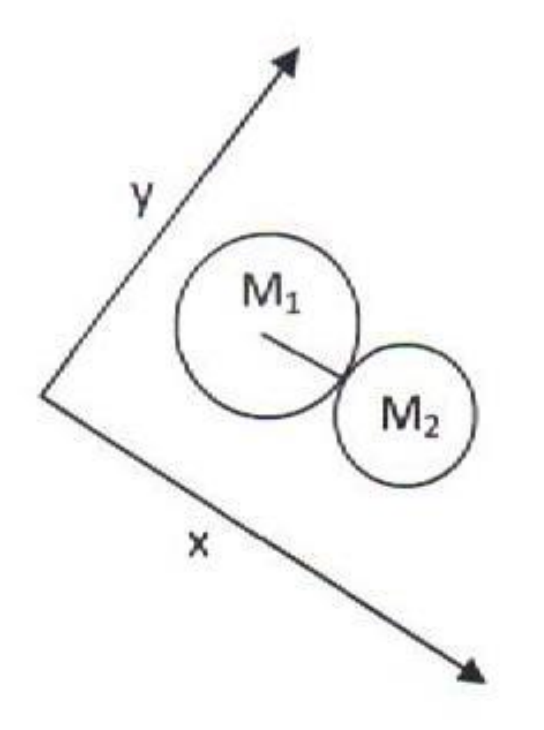


$$v_{1xr} = v_1(cos(\theta_1 - \phi))$$

$$v_{1yr} = v_1(sin(\theta_1 - \phi))$$

$$v_{2xr} = v_2(cos(\theta_2 - \phi))$$

$$v_{2yr} = v_2(sin(\theta_2 - \phi))$$



The Math
Elastic Collisions

- The axes are now rotated in such a way that the collision is only in the xr direction. Since the collision is now 1-dimensional, we can use the conservation of momentum equation for a perfectly elastic collision

$$v_{1f} = \frac{v_1(M_1 - M_2) + 2M_2v_2}{M_1 + M_2}$$

- If you are unfamiliar with this equation, consider the following examples to gain some intuition:
  - if M1 >> M2, v1 will be unaffected
  - if M1 << M2, v1 will become −v1

The Math
Elastic Collisions

- Combining equations:

$$v_{1xr} = v_1(cos(\theta_1 - \phi))$$

$$v_{1yr} = v_1(sin(\theta_1 - \phi))$$

$$v_{2xr} = v_2(cos(\theta_2 - \phi))$$

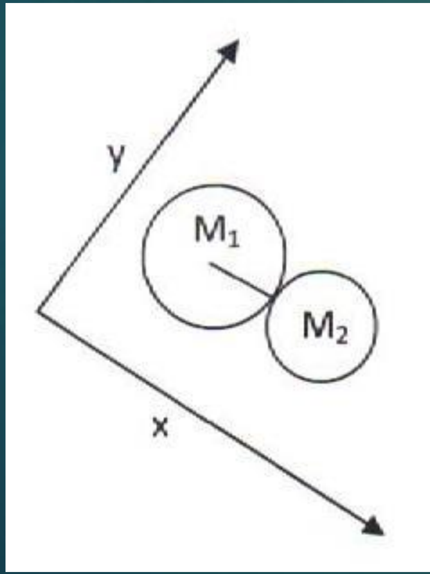
$$v_{2yr} = v_2(sin(\theta_2 - \phi))$$

$$v_{1xrf} = \frac{v_1(cos(\theta_1 - \phi))(M_1 - M_2) + 2M_2v_2(cos(\theta_2 - \phi))}{M_1 + M_2}$$

$$v_{1f} = \frac{v_1(M_1 - M_2) + 2M_2v_2}{M_1 + M_2}$$

The Math
Elastic Collisions

- Now we must calculate the x- and y-velocities relative to the natural(non- rotated) axes. We do this using the contact angle, φ.
- We add π/2 when using the y-velocity because we are moving along the rotated y-axis and our φ is relative to the rotated x-axis

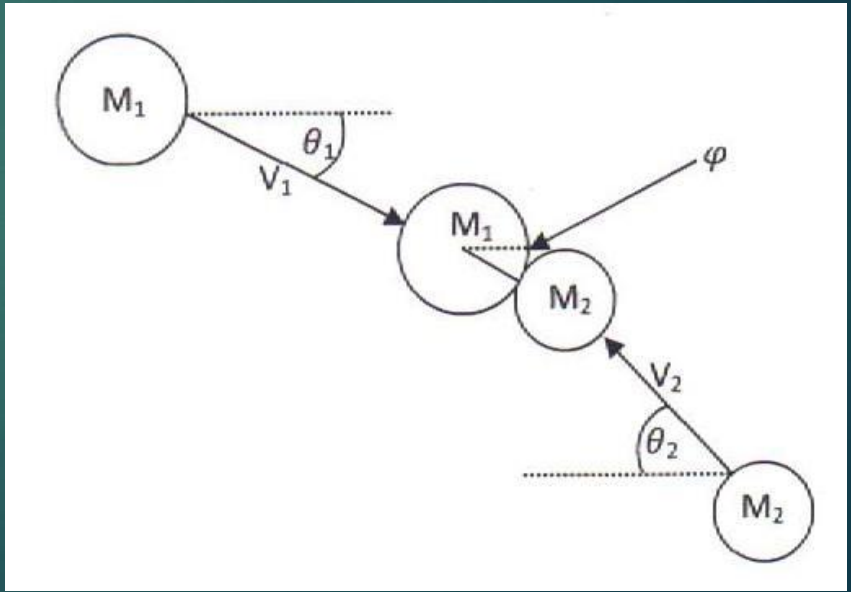


$$v_{1xf} = v_{1xrf}cos(\phi) + v_{1yr}cos(\phi + \frac{\pi}{2})$$

$$v_{1yf} = v_{1xrf}sin(\phi) + v_{1yr}sin(\phi + \frac{\pi}{2})$$

$$v_{2xf} = v_{2xrf}cos(\phi) + v_{2yr}cos(\phi + \frac{\pi}{2})$$

$$v_{2yf} = v_{2xrf}sin(\phi) + v_{2yr}sin(\phi + \frac{\pi}{2})$$



The Math
Elastic Collisions

- Combining equations once again leaves us with a simple way to calculate the final velocities of two colliding balls that very accurately mirrors reality

$$v_{1xf} = v_{1xrf}cos(\phi) + v_{1yr}cos(\phi + \frac{\pi}{2})$$

$$v_{1yf} = v_{1xrf}sin(\phi) + v_{1yr}sin(\phi + \frac{\pi}{2})$$

$$v_{2xf} = v_{2xrf}cos(\phi) + v_{2yr}cos(\phi + \frac{\pi}{2})$$

$$v_{2yf} = v_{2xrf}sin(\phi) + v_{2yr}sin(\phi + \frac{\pi}{2})$$

$$v_{1yr} = v_1(sin(\theta_1 - \phi))$$

$$v_{2yr} = v_2(sin(\theta_2 - \phi))$$

$$v_{1xrf} = \frac{v_1(cos(\theta_1 - \phi))(M_1 - M_2) + 2M_2v_2(cos(\theta_2 - \phi))}{M_1 + M_2}$$

$$v_{2xrf} = \frac{v_2(cos(\theta_2 - \phi))(M_2 - M_1) + 2M_1v_1(cos(\theta_1 - \phi))}{M_1 + M_2}$$

$$v_{1xf} = \frac{v_1(cos(\theta_1 - \phi))(M_1 - M_2) + 2M_2v_2(cos(\theta_2 - \phi))}{M_1 + M_2}cos(\phi) + v_1(sin(\theta_1 - \phi))cos(\phi + \frac{\pi}{2})$$

$$v_{1yf} = \frac{v_1(cos(\theta_1 - \phi))(M_1 - M_2) + 2M_2v_2(cos(\theta_2 - \phi))}{M_1 + M_2}sin(\phi) + v_1(sin(\theta_1 - \phi))sin(\phi + \frac{\pi}{2})$$

$$v_{2xf} = \frac{v_2(cos(\theta_2 - \phi))(M_2 - M_1) + 2M_1v_1(cos(\theta_1 - \phi))}{M_1 + M_2}cos(\phi) + v_2(sin(\theta_2 - \phi))cos(\phi + \frac{\pi}{2})$$

$$v_{2yf} = \frac{v_2(cos(\theta_2 - \phi))(M_2 - M_1) + 2M_1v_1(cos(\theta_1 - \phi))}{M_1 + M_2}sin(\phi) + v_2(sin(\theta_2 - \phi))sin(\phi + \frac{\pi}{2})$$

References

- Elastic Collisions. William Craver. URL: http://williamecraver.wix.com/elastic- equations [cited 11/20/2013].