Analysis of a Simple Two-Dimensional Double Pendulum in VPython

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The purpose of this project is to explore the motion of a double pendulum in two dimensions, beginning with a situation with equivalent lengths and masses on each pendulum and potentially working toward more complicated cases.

For any two-dimensional rigid simple double pendulum with massless rods and point masses in uniform gravity g, the Lagrangian is given by

$$L = \frac{(m_1 + m_2)l_1^2\dot{\theta}_1^2}{2} + \frac{m_2 l_2^2 \dot{\theta}_2^2}{2} + m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2) + (m_1 + m_2) g l_1 \cos(\theta_1) + m_2 g l_2 \cos(\theta_2)$$

where *m* and *l* give mass and length, respectively, θ represents the angle of the pendulum from the vertical and the subscript 1 refers to the top pendulum while the subscript 2 refers to the bottom pendulum. The motion can then be described by numerically. One must perform the Legendre Transform to arrive at:

$$H(\theta_1, \theta_2, p_1, p_2) = \sum_{i=1}^{2} \dot{\theta}_i \, p_i - L(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2)$$

where *H* is the Hamiltonian and

$$p_i = \frac{\partial L}{\partial \dot{\theta}_i}$$

Then one can arrive at the necessary equations to solve numerically by the fourth-order Runge-Kutta method.

I will use VPython for graphical programming. I plan to edit Bruce Sherwood's VPython for accuracy at high energies by implementing the Runge-Kutta method. I intend to create an interface with which the user can choose the ratio of masses and the initial angles. I would also like to display in separate colors the path that the second pendulum follows. I will endeavor to develop a graphical representation of the phase portraits for double pendulum by tracking its motion in real time.

Timeline:

November 3rd, 2013- Have altered Bruce Sherwood's program to display a simple double pendulum rather than a compound double pendulum, including a curve to map the lower pendulum's motion

November 10th, 2013- Have created a manner in which the user can alter the mass ratio and the initial angles of the system

November 17th, 2013- Have gleaned sufficient understanding of LaTeX to recreate this document and design my webpage

December 1st, 2013- Have implemented the Runge-Kutta method to accurately display the pendulum at high energies

December 8th, 2013- If possible, have devised and implemented a way to display the phase portraits of the system