A Lesson on Recursion with a Modified Menger Sponge

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Abstract

The Menger Sponge is a fractal curve and a 3-dimentional representation of the Cantor Set. The objective of this project is to represent the Menger Sponge in 3 dimensions with a Python/OpenGL program, but the extra twist involves a technique called selective suppression. This process will allow the program to randomly terminate its recursive sequence so that the Menger Sponge can be observed in different stages of recursive development.

1 Introduction

Since my high schoool years I have been intrigued by visual depictions of fractal curves. I find fractals like the Mandelbrot Set, Sierpinski Gasket, and the Pythagorean Tree to be beautiful displays of mathematical imagery. After viewing the Cantor Set (and it's 3D depictions, Cantor Dust and the Menger Sponge), I decided that I would seek to create one of these fractal curves in my own program, for only a few iterations. The Menger Sponge was first described by mathematician Karl Menger in 1926 as a 3D depiction of the Sierpinski Carpet (which itself is a 2D depiction of the one-dimentional Cantor Set). It has the unusual property of an infinite surface area while also having a volume of zero. Physically, one can describe the Menger Sponge of 0 iterations as a simple cube. At one iteration, you remove a rectangular prism extending from the middle of each of the faces of the cube. At two iterations, you perform the same process on each of the 20 cubes in the set. At three iterations, you perform the process once again on each cube.

2 Project Description

Currently I have a program named 'szgcube.py' that correctly depicts the Menger Sponge for 1-3 iterations using a recursive method and a check variable 'n'. The final project will add another check based on a random number generator that will stop the recursive process if the random number is within a certain threshold. Ultimately, the project should depict a modified Menger Sponge with some sections displayed at n=1 iterations, and other sections at n=2 or n=3 iterations. This will hopefully create a unique and aesthetically pleasing shape that demonstrates a snapshot of recursion.

I have been working with matrix transformations using glTranslatef() and glScalef(), among other matrix methods, to develop two drawing methods (described later) that are the fundamental components of this project. The Menger Sponge in the program is drawn entirely with individual cubes, and these cubes were created with triangular strips through OpenGL's default methods.

The primary programs I have edited to create the source code for my project is the torus2skel program that renders a Torus in Python/OpenGL.

3 Goals and Accomplishments

- ACCOMPLISHED: Learn basic HTML tags and create an HTML webpage for this project (developed 10/25/12) using much of the information found in this document.
- ACCOMPLISHED: Develop a drawcube() method in my program cube.py (Python/OpenGL) with Matt Hoffman's help. This method simply creates a colorful cube with adjustable parameters for position and size scaling (using matrix transformations)
- ACCOMPLISHED: Develop a drawMenger() method that calls itself multiple times (recursion) in order to set the positions for the cubes in the Menger Sponge. Upon the last iteration, the method calls drawcube() instead of itself in order to fill the space with cubes of appropriate size.
- ACCOMPLISHED: Edit szgtorus.py into szgcube.py that calls methods from cube.py to generate a Menger Sponge for 1-3 iterations. This file has successfully run in the CUBE south of campus.
- Need to develop a way of having the drawMenger() method check for a parameter through a random number generator to see if it should iterate on a certain

cube. This will allow for the selective suppression to take place, resulting in some cubes iterating the method ("dividing") while others staying the same.

- Potential considerations (if the previous task proves not to be a significant challenge): allowing for shadow effects and alternate color schemes of the cube, perhaps altering the particle patterns on the Menger Sponge's surface so that I can put other objects on it.