PredPrey: Predator Prey Cellular Automaton in PyOpenGL

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Abstract. The purpose of this project is to understand the interactions between a prey and a predator by using a cellular automaton. The Lotka-Volterra differential equations show the relationship between the predator and prey. This project uses a two dimensional cellular automaton made with OpenGL with rules that are independent of the Lotka-Volterra equations. Still, the equilibriums that result from these rules are relatively consistent with the Lotka-Volterra Equations.

Lotka-volterra equations. The Lotka- Volterra differential equations

$$\frac{dx}{dt} = x(\alpha - \beta y) \qquad \frac{dy}{dt} = -y(\gamma - \delta x)$$

show how the prey and predator change in relationship to each other respectively. In these equations x represents the number of prey and y represents the number of the predators. $\frac{dx}{dt}$ and $\frac{dy}{dt}$ represent the change in size of the two populations over time. α, β, γ, and δ are parameters representing the interactions of the two species and their environment.

Distributing out these formulas show exactly what they mean.

$$\frac{dx}{dt} = x\alpha - x\beta y \qquad \frac{dy}{dt} = \delta xy - \delta y$$

The prey population is shown to grow at a rate of αx and at the same time decrease by the rate of predation, designated by βxy. The predator population is shown to grow at a rate of δxy and decrease at the rate of natural death, γy.

By solving these differential equations for zero we can find the two instances of population equilibrium.

$$y = 0, x = 0 \qquad\qquad y = \frac{\alpha}{\beta}, x = \frac{\gamma}{\delta}$$

The first of these solutions occurs when both predator and prey are extinct or when the predators are extinct and the prey grow till they reach their carrying capacity. We are interested in the second instance where the functions behave in a cyclical fashion, revolving around the values $y = \frac{\alpha}{\beta}, x = \frac{\gamma}{\delta}$. This cyclical behavior is known to become very interesting and chaotic in the third dimension.

Cellular Automata. By using a simple cellular automata where each cell (organism) follows a simple set of proximity-based rules, this project proceeds to show that this Lotka-Volterra equilibrium will be maintained.

The cellular automata for this project consists of three main parts:

1. A square grid framework of "cells" that actually consists of many large points plotted in a grid pattern.
2. Different physical representations in each square for the prey, predator, and bare ground.
3. A set of rules that each cell will obey designed to imitate the behavior of the prey, predator, or bare ground.

The goal is that the growth of each of the species portrayed in this cellular automata will be consistent with the growth predicted by the Lotka-Volterra differential equations. This would be a very interesting result in that it shows how the sum of the behaviors of the individuals can result an interesting behavior in itself, very similar to how nature functions.

The Grid. By using some very simple algebra, the grid was made to be universal to any side length for the square box. The user just changes the number of columns of cells they would like, and the window and the program will automatically resize the cells and their spacing so that they fit the window properly.

The Cells. The cells consist of the different colored points with locations defined by the grid mentioned previously. The predators are represented by red cells, the prey are represented by green cells, and the empty areas are represented by black cells. Numbers of 0, 1, and 2 were assigned to the predators, prey, and empty cells respectively and stored in an array so that they identity of any particular cell can be recalled for the calculation of the next generation.

Generation 0. To ensure that each running of the program yields different results a python random function was used to randomly distribute predators, prey, and bare ground at the start of the program.

Subsequent Generations. The future generations are decided by the neighbors of the cells. There are two possible neighborhoods to use in a cellular automaton: the Von Neumann and Moore neighborhoods. The Von Neumann neighborhood consists of the four neighbors in the cardinal directions: up, down, left, and right. The Moore neighborhood uses all eight squares that border a given cell. This cellular automaton uses the Moore neighborhood because it allows for more flexibility in the cell rules.

When the neighborhoods are applied to the cells at the edges of the window problems occur because the cells in these areas do not have as many neighboring cells as the squares in the center of the window. To solve this problem two things can be done; either these cells must have different rule definitions, or the window can be wrapped top to bottom and left to right. This program uses window wrapping because it add the effect of a three dimensional torus surface.

Next, the program recalls the numbers of 0 and 1 to count the number of predators and prey around a cell. Then depending on whether the cell itself is a predator, prey, or empty ground, the program will go through a set of rules that depend on the number of predators and prey around the cell and decide its fate for the next generation.

Program Results The first programs show the various solutions of the Lotka-Volterra equations. The first solution $y = 0, x = 0$ can produce two possible results: the prey die and the predators quickly follow, and only the predators die. These can be shown in the cellular automaton by altering the rules for the predators and prey to adjust them to their desired strength or weakness.

Steady State. The interesting program is when the rules are altered to make the predators and prey live together in a steady-state automaton. Here the ratio of predators to prey remain around 3.0 for a seemingly infinite number of generations.

Conclusions. A cellular automaton can be a great way to model complex behaviors. By making cell rules to mimic the behavior of organisms in nature this program was successfully able to show that the Lotka-Volterra equations are exhibited in predator prey relationships.

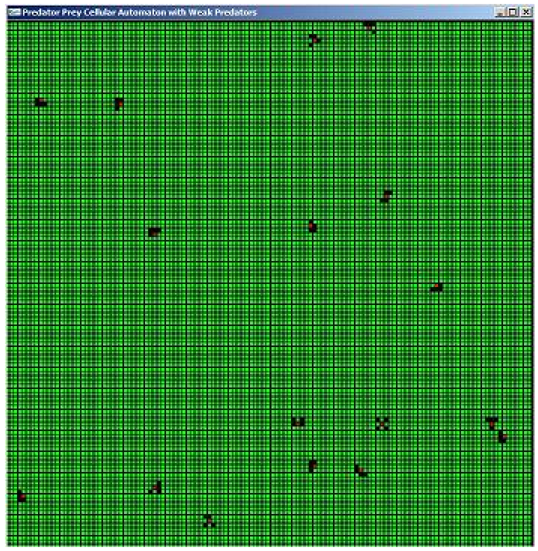

Figure 1: Only the predators are extinct

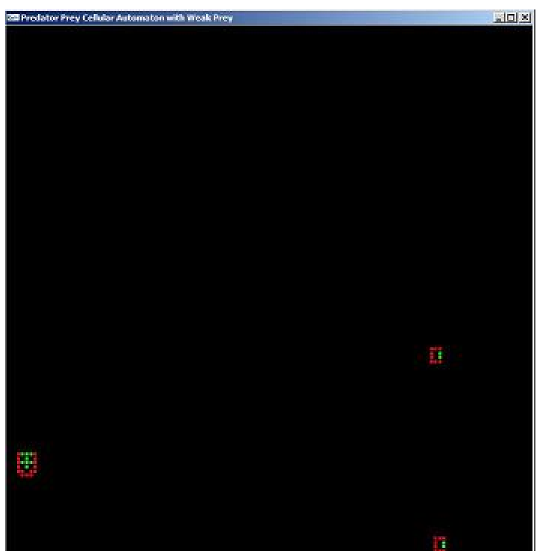

Figure 2: Both the predators and prey are extinct

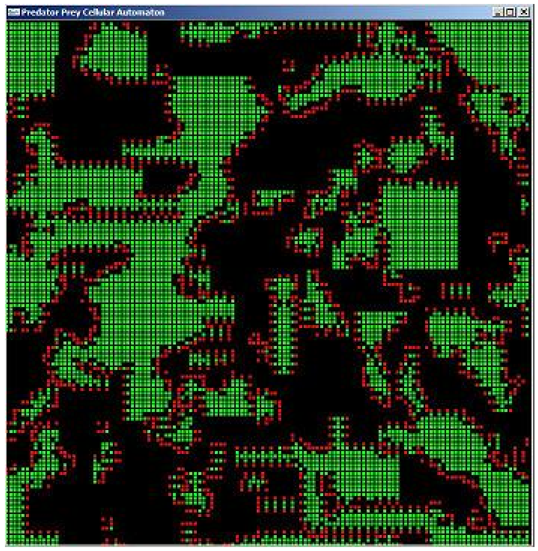

Figure 3: When predators and prey reach a steady state many interesting phenomena can be observed.

Figure 4: When a factor of overpopulation is added to the predators large patches of death appear within the colonies. This shows that the prey are healthiest when there are predators near.