Cellular-Automata Supercomputers for Fluid-Dynamics Modeling

Norman Margolus, Tommaso Toffoli, and Gérard Vichniac
Laboratory for Computer Science, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
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We report recent developments in the modeling of fluid dynamics, and give experimental results (including dynamical exponents) obtained with cellular automata machines. Because of their locality and uniformity, cellular automata lend themselves to an extremely efficient physical realization; with a suitable architecture, an amount of hardware resources comparable to that of a home computer can achieve (in the simulation of cellular automata) the performance of a conventional supercomputer.

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A cellular automaton is a discrete dynamical system consisting of finite-state variables, or *cells*, arranged on a uniform grid. The overall dynamics is specified by a finite *rule*, by which at every time step each cell computes its new state from the current state of its neighborhood. Surprisingly enough, cellular automata can faithfully model *continuum* systems such as fluids; unlike differential equations, they can be realized *exactly* by digital hardware.

Modeling with cellular automata is poorly supported by conventional scientific computers, whose architecture is optimized for the arithmetic treatment of continuum models. With a more appropriate architecture one can easily gain a performance factor of at least 10 000 in the simulation of cellular automata; this gain is of such magnitude that new classes of conceptual models have become computationally accessible.

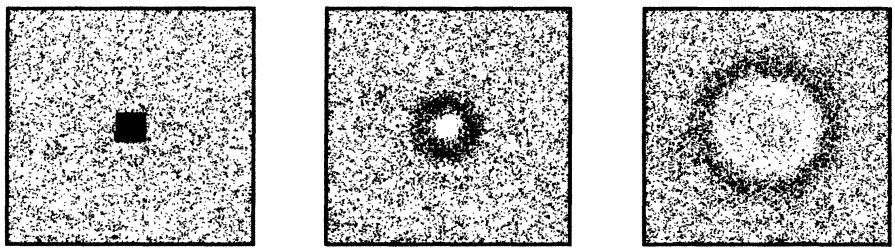
The idea of using discrete lattices for modeling physical phenomena is not new. However, recent theoretical and technological developments have turned models based specifically on cellular automata into practical computational tools. On one hand, methods have been found for constructing cellular automata that are microscopically reversible (and thus support a realistic thermodynamics), obey exact conservation laws, and model continuum phenomena.[1-10] On the other hand, general-purpose machines well suited to such fine-grained modeling are becoming generally available.[10-17]

Hydrodynamic modeling.—Differential equations such as the Navier-Stokes equation capture important macroscopic aspects of fluid dynamics; however, what one implements on a digital computer is not the equation itself, but a finitary model obtained from it by truncation and roundoff.

It is possible to arrive at an analogous macrodynamics starting directly from a discrete microscopic model—a cellular-automaton idealization of the motion and collisions of individual particles. Models of this kind can give rise to the Navier-Stokes equation in the macroscopic limit,[9] as had been indicated as early as 1973 by Pomeau and co-workers[1,2] in a theoretical analysis of a lattice-gas model (hereafter, HPP gas).

Frames (a), (b), and (c) of Fig. 1 are taken directly from the display screen of CAM-6,[11] a cellular-automata machine. They show the evolution of an HPP gas consisting of 2^{16} sites (256×256) each of which can hold up to four particles (one traveling in each of four allowed directions). The evolution rule is simply that particles travel straight at unit velocity (one cell per time step) unless exactly two particles collide head on, in which case they scatter at right angles.

The initial state (a) was constructed with a bit occu-



(a) (b) (c)

FIG. 1. Wave propagation in the HPP gas. To enhance contrast, only sites that contain either three or four particles are shown.

pancy of 50% except in the middle, where we have a block of 4096 particles (100% occupancy in an area of 32×32 sites). Thus, the simulation involves about 130 000 particles. Figures 1(b) and 1(c) show the start that thirty and ninety steps–0.5 and 1.5 seconds at CAM's rate of sixty frame updates per second. Despite the fact that particles travel in only four directions the wave is circular, and moves at a speed that agrees with the theoretically predicted[2] value of $1/\sqrt{2}$.

The HPP rule is exactly reversible. To go back in time from frame Fig. 1(c) one need only transform the state of every cell so as to interchange the "up" and "down" particle information, and similarly for "right" and "left." If one then proceeds with the same rule the simulation will retrace its steps back to Fig. 1(a).

Boundary conditions such as sources, sinks, and obstacles of any shape are introduced by use of additional bits of state at each site to mark selected areas, and extension of the rule so as to take the values of these bits into account.[10]

Time correlation functions.—We have measured the time autocorrelation function for velocity at a given site,

$$\nu(t)=\lim_{T\to\infty}\frac{1}{TNQ}\sum_{t'=1}^{T}[a_{ij}^{q}(\phi^{t+t'}C)-\bar{a}]\times[a_{ij}^{q}(\phi^{t'}C)-\bar{a}],$$

where C is an initial configuration, ϕ is the transition rule (such as HPP), $a_{ij}^{q}(C)$ has a value of 1 or 0 depending on whether or not there is a particle moving in the q direction at the (ij) site in the configuration C, $\bar{a}$ is the average particle occupancy (per site) for each direction, and there is an implied summation over all N sites and all Q allowed directions.

The actual correlation experiments were performed with use of a cellular-automata realization of the HPP rule[18] that spreads each site over four one-bit cells.[4,10] Figure 2, curve a, shows the measured values of $\nu(t)$ for the HPP model, with $T=2^{13}$ and a density of $\bar{a}=\frac{1}{4}$. Finite-size effects show up past $t=256$ (the space is 256×256 and has periodic boundary conditions), but already before that point the predicted[19] asymptotic exponent of $-\frac{2}{3}$ is attained within $\pm3\%$. The same exponent was obtained for $\bar{a}=\frac{1}{6}$, $\bar{a}=\frac{1}{8}$.

The exponent $-\frac{2}{3}$, which is characteristic of one-dimensional gases,[19] arises from extra conservations (momentum on each row and column). The "TM" gas[10,20]—a similar model which also uses only four directions—avoids these extra conservations by having collisions occur with a nonzero impact parameter (resulting in right-angle scattering from two adjacent rows to two adjacent columns, or vice versa). Results for this gas are shown in Fig. 2, curve b, for the same density $\bar{a}$ as for HPP but (to improve the statistics) for $T=2^{16}$; note that the asymptotic exponent for this gas

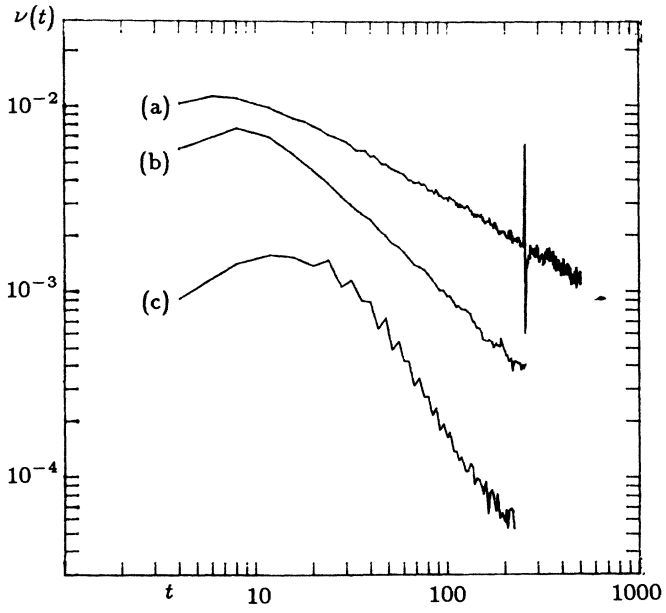


FIG. 2. Time correlation function, $\nu(t)$, for (curve a) the HPP gas, (curve b) the TM gas, and (curve c) the FHP gas; $\bar{a}=\frac{1}{4}$ in all cases.

is close to -1, characteristic of a true two-dimensional gas ($\nu\sim t^{-d/2}$, for $d\geq2$).[21] The same exponent was obtained for $\bar{a}=\frac{1}{6}$, $\bar{a}=\frac{1}{8}$.

As is noted in Ref. 9, a shadow of the $\pi/2$ rotational symmetry persists in the macroscopic behavior of the HPP gas (and possibly also in TM). A more refined hydrodynamical model (the "FHP gas"[9,22]), which uses a hexagonal grid, avoids this problem.[23] Figure 2, curve c, shows the behavior of $\nu(t)$ for this gas, with $T=2^{17}$ and again with the density $\bar{a}=\frac{1}{4}$. The asymptotic exponent appears to be close to -2, which is surprisingly large and calls for a theoretical explanation—the same exponent was obtained for $\bar{a}=\frac{1}{6}$, $\bar{a}=\frac{1}{8}$. As above, this experiment used an implementation in which collision sites are spread out—in this case over four two-bit cells.[10]

Hardware.—These experiments were performed with a machine (CAM-6[11]) containing an amount of digital logic comparable to that of a home computer. This machine achieves, in the simulation of cellular automata, a performance comparable to that of a Cray-1; several modules can be ganged together—with a proportionate increase in performance—for larger two-dimensional arrays or for three-dimensional simulations.

To achieve maximum speed, the rule is internally stored as a lookup table. The rule as written by the user consists of a few lines describing in a high-level language how the new value of a cell depends on the current values of its neighbors. The problem of translating such a description into an appropriate look-

up table is taken care of by the machine's designers; since the efficiency of this compilation process in no way affects the speed of the simulation, one's full attention can be kept on conceptual issues.

The above performance is achieved with a dedicated architecture but conventional circuitry and components. In this architecture two-dimensional planes are processed *serially* (with a substantial amount of pipelining); a third dimension is achieved by stacking planes and operating them in parallel.[15] Since sites in each plane are updated one by one, and corresponding sites in adjacent planes are handled at the same time in a "lockstep" fashion, communication between planes entails a few wires rather than the millions of physical interconnections required by a fully parallel implementation. This approach makes extensive simulation of three-dimensional models of hydrodynamics immediately practical.

One may simultaneously run (or different planes of the same machine) two copies of the same system that are identical except for a given spatial or temporal offset between them. Since sites are processed serially, correlations between corresponding sites are easily detected and accumulated "on the fly," thus eliminating the need for storage- and computation-intensive postprocessing. This is, in fact, how the time-correlation experiments presented in this paper were conducted.

Conclusions.—Cellular-automata machines are well suited to a large class of computational models of physics having both theoretical and practical interest. The performance they offer provides strong encouragement for the development of models of this kind; conversely, the usefulness of these models will stimulate technology to provide further performance in this direction.

Because of the speed-of-light constraint, locality of interconnection is an important advantage of the cellular-automaton paradigm. A fully parallel implementation of specific two-dimensional cellular automata having 10^{12} sites and an update cycle of 100 ps for the *whole array* will be feasible in one decade and within easy reach in two; one using 10^{16} sites (the Avogadro number "in two dimensions") is not inconceivable. Thus, we can look forward to computational tools that *directly span the gap between the microscopic and the macroscopic world.*

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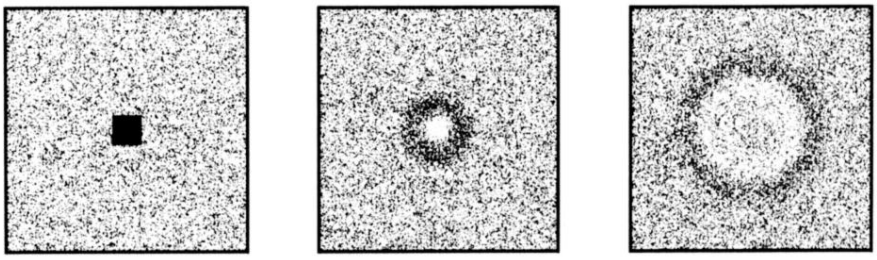

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