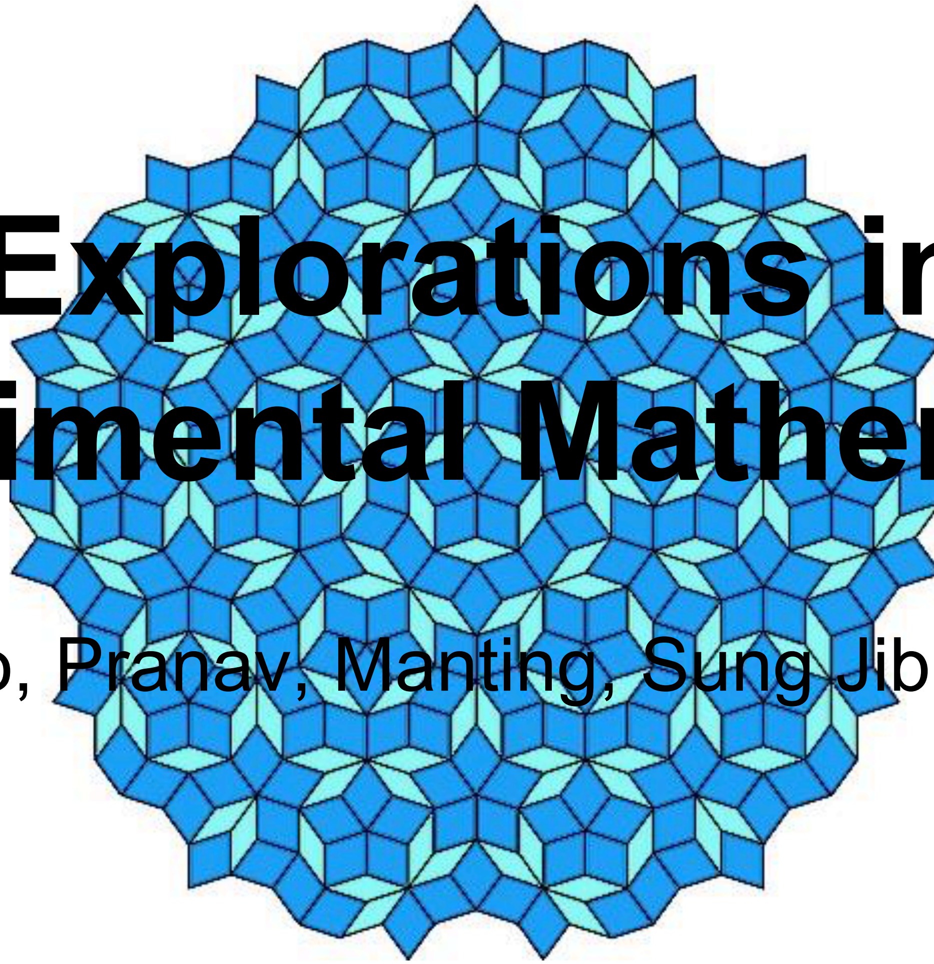
Explorations in Experimental Mathematics



Tejo, Pranav, Manting, Sung Jib, Lisa

Experimental Mathematics

"Experimental mathematics is an approach in which computation is used to investigate mathematical structures and identify their fundamental properties and patterns"

For this project, python code was developed to explore cobweb plots of the logistic chaos equation

Logistic chaos equation: $y=4*a*(1-x)*x$

Process of Coding

Our group attended Stefan Klajbor’s Python workshop, which taught us the basics to code this program

We have python versions using the TkInter and Turtle libraries

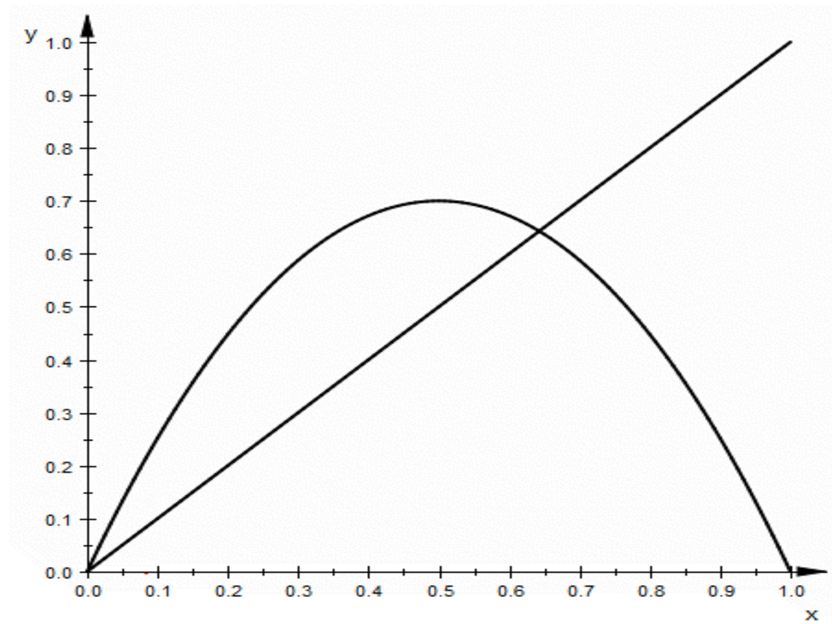
A Javascript version is being developed by our group as well

Cobweb plots AKA Verhulst Diagram

Shows the behavior of chaotic orbits in dynamical systems

A cobweb plot repeats the following steps during each iteration:

1. Find a starting point on the curve (x,f(x))
2. Plot a horizontal line from this point to the diagonal line y=x
   a. The coordinates of this point are (f(x), f(x))
3. Plot a vertical line from this point to the curve
   a. The coordinates of this point are (f(x), f(f(x)))



Behaviors of Cobweb Plot

Using Experimental Mathematics, we looked into the behaviors of the cobweb plot with the function: $y=4*a*(1-x)*x$

Using this approach, we have discovered four distinct behaviors:

1. **$a<.25$** : graph dies (converges to 0)

2. **$.25<a<.5$**: converges to nonzero point

3. **$a>.5$**:

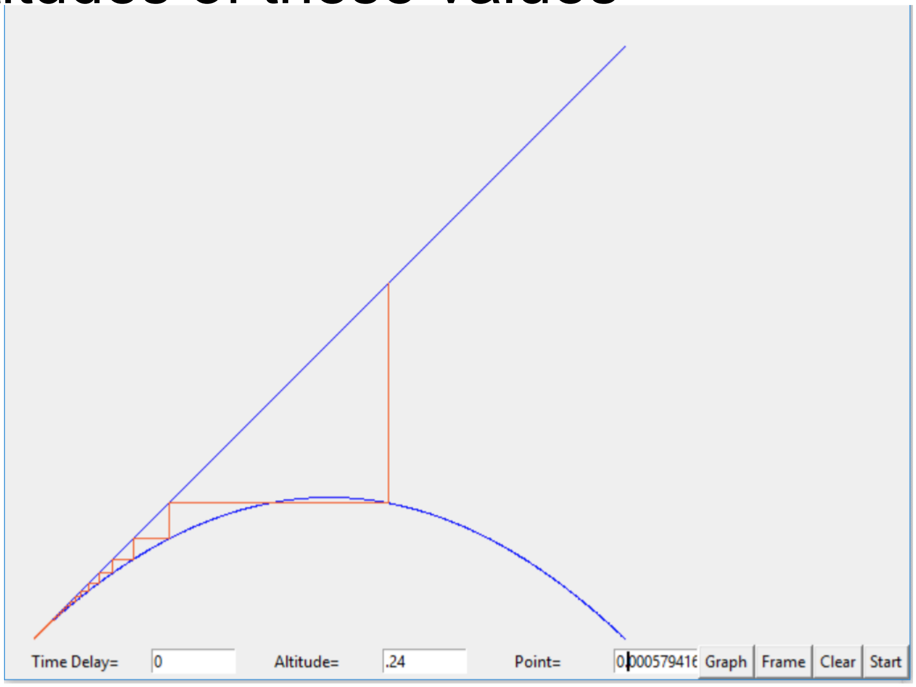
   $.5<a<.96$: Period 2

   $a= .96$: Period 3

4. **$a> .96$**: Chaotic

Behavior 1

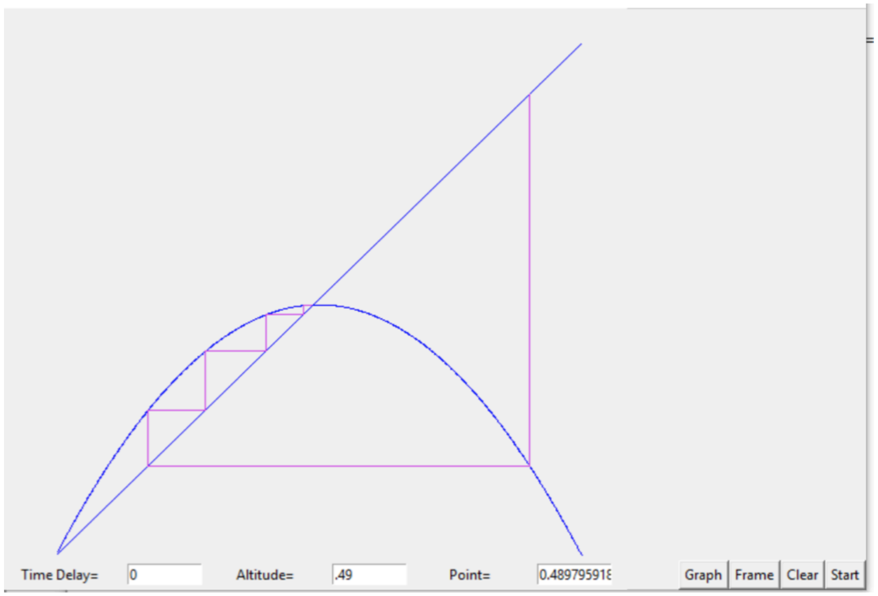
a<.25
The graph converges to 0 for altitudes of these values



Behavior 2

.25<a<.5

The graph converges to a single, nonzero point for altitudes of this value

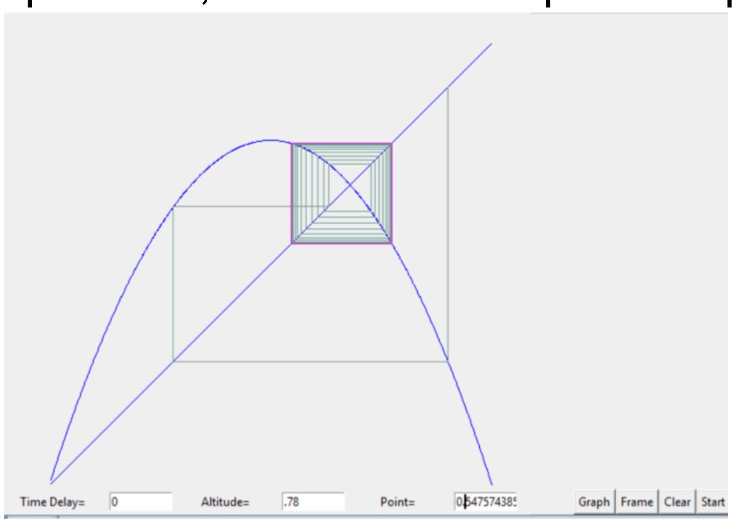


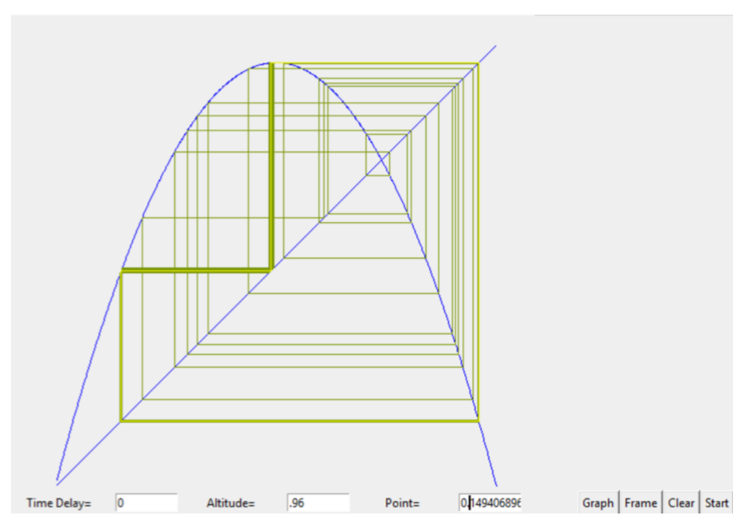
Behavior 3

.5<a<.96

The graph is multi-period

Sharkovsky’s Theorem: if a continuous real function has a periodic point with period 3, then there is a periodic point of period n for every integer n.

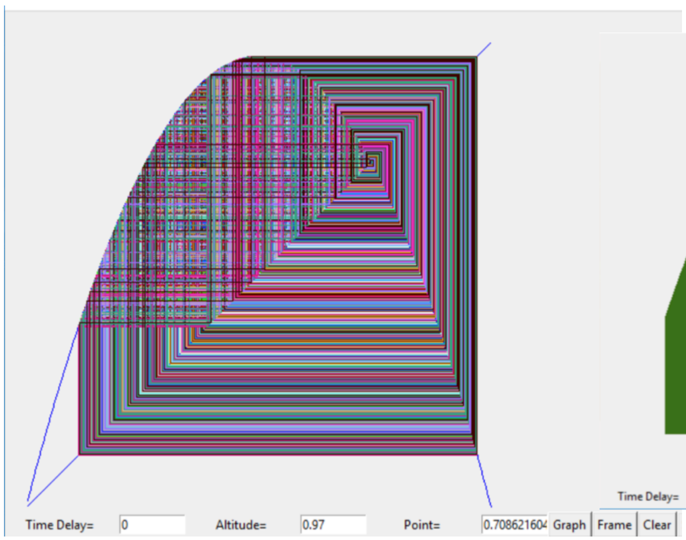


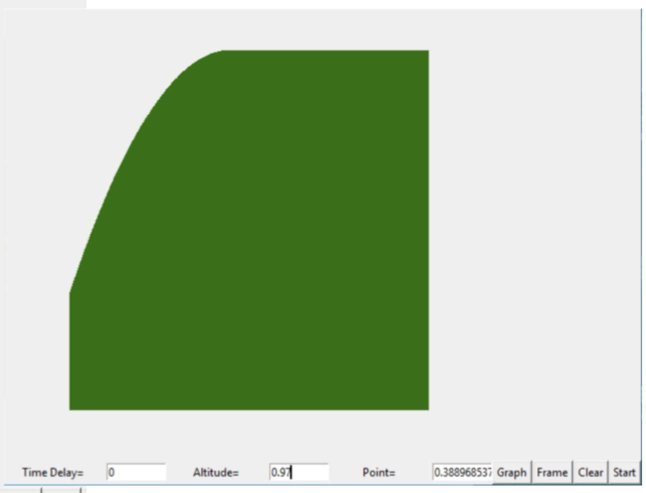


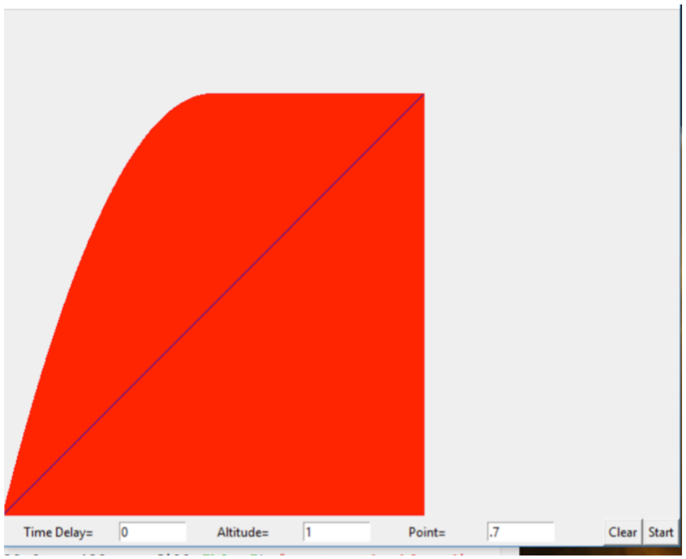
Behavior 4

For altitudes greater than 0.96 (a>0.96) the cobweb plot does not converge to any number of points

The graphs below show how after many iterations, the cobweb plot displays chaotic behavior by filling the visible portion of the graph







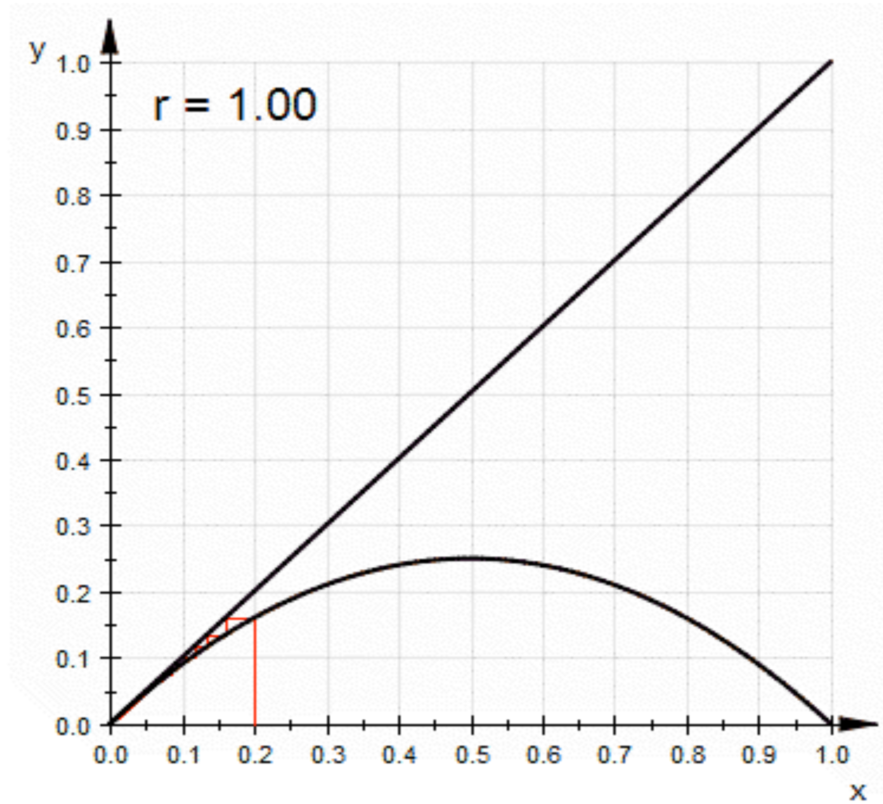
Demonstration

Sample altitudes

- a= .2
- a= .45
- a= .77
- a=.96
- a=1

All Together

- The graph on the right show all the behaviors as the altitude, a, goes from .25 to 1
- Note that r = 4*a



Stability of 2D Quasicrystals

Using computer animations, we can apply the experimental mathematics approach to analyze the rigidity of Quasicrystals in 2D and 3D

Wester 2D game

Wobble.html

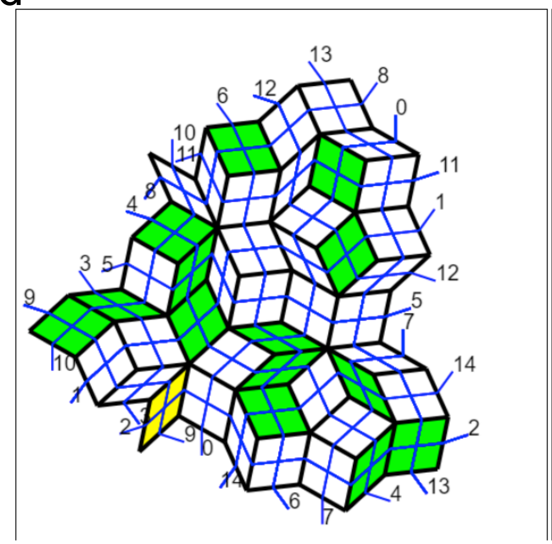
Wester 2D Game

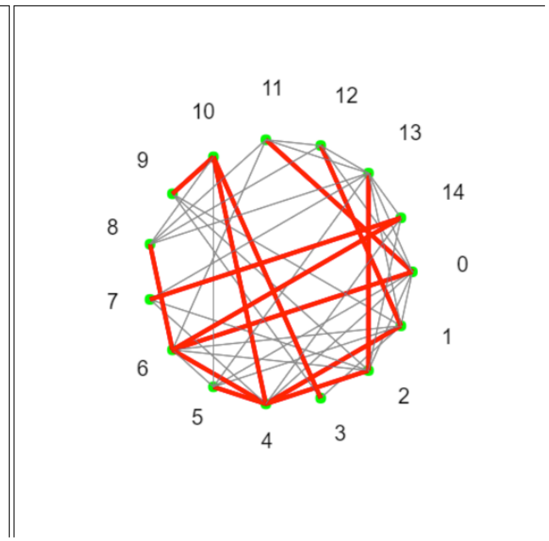
Using the theorems of Wester, Baglivo, and Graver

Bracing the the rhombi corresponding to the edges of the spanning, connected subgraph makes it rigid

Proof of the 2D case

- The following is a possible solution for rigidity of the carpet
- Note the corresponding graph





Wobble.html

Use this program to extend theorems for the 2D case to the 3D case for blocks

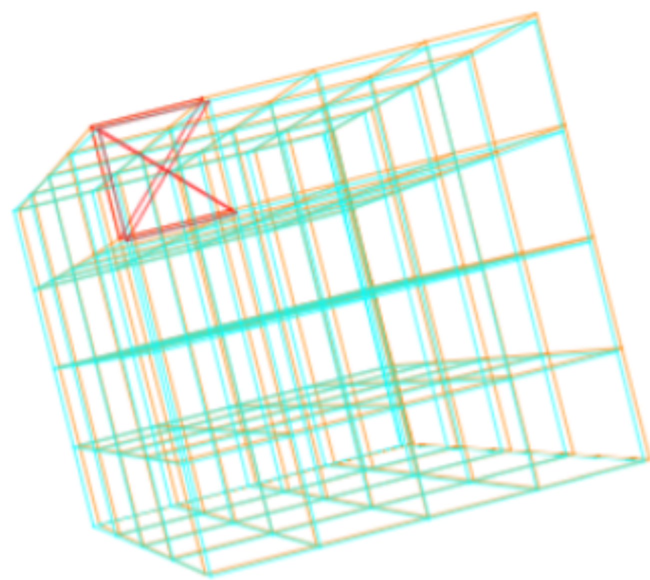
Eventually plan to extend the 3D grid to a 3D rhombic cluster

More complex shifts

Using :

Tunnel conjecture

Shadow conjecture



Bibliography

"Experimental Mathematics." *Experimental Mathematics -- from Wolfram MathWorld*. N.p., n.d. Web. 08 Mar. 2017.

"Cobweb plot." *Wikipedia*. Wikimedia Foundation, 18 Feb. 2017. Web. 08 Mar. 2017.

Stoop, Ruedi; Steeb, Willi-Hans (2006). *Berechenbares Chaos in dynamischen Systemen*[*Computable Chaos in dynamic systems*] (in German). Birkhäuser Basel. p. 8. ISBN 978-3-7643-7551-5.

Stoop, Ruedi; Steeb, Willi-Hans (2006). *Berechenbares Chaos in dynamischen Systemen*[*Computable Chaos in dynamic systems*] (in German). Birkhäuser Basel. p. 8. ISBN 978-3-7643-7551-5.

Wobble.html: **IGL team: Dan Pugliese, Chong Han, Joe Zeller, Zach Miksis ; Mentors: George Francis, Eliana Duarte**

Wester game: **IGL team: Chong Han, Alec Mori, Dan Publiese, Joe Zeller, Zach Miksis ;Mentors: Eliana Duarte, George Francis**